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Date: 12/1/2025

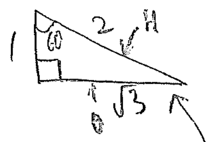
Math 10/11 Honors HW Section 4.1 Review on Basic Trigonometry:

1. When I use regular trigonometric functions like sine, cosine, and tangent, does it only work for right triangles? Or can I use it for all different types of triangles?

   works for all triangles (sine law & cosine law can be used)

2. When I sine an angle like 60°, it gives me a value like 0.866025403. What does this number represent?

   $S \frac{O}{H}$

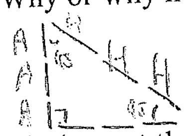
   

   30-60-90 △

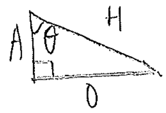
   The $\frac{\text{opposite side}}{\text{Hypotenuse}}$ $\sin 60^\circ \rightarrow \frac{\sqrt{3}}{2} = 0.866025403$

3. There are 2 similar right triangles where one is three times bigger than the other. They are both $45^\circ - 45^\circ - 90^\circ$ triangles. If I cosine the 45° in the smaller triangle, will it give me the same value when I cosine the 45° of the bigger triangle? Why or why not? $C \frac{A}{H}$

   Yes, same value $\frac{A}{H}$

    $\rightarrow$  $\frac{3A}{3H} \rightarrow \frac{A}{H}$

4. When I cosine or sine any angle in a right triangle (except the 90°) will I ever get a value greater than 1? Why or why not?

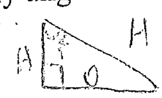
   

   Hypotenuse is always $\geq$ opposite side

   $S \frac{O}{H} \rightarrow \sin\theta$ must be ≥ 1

5. When I use tangent on any angle in a right triangle (except the 90°) will I ever get a value greater than 1? Why or why not?

   $T \frac{O}{A}$ Yes.

   Opposite can be longer than adjacent side.

6. What does the inverse trigonometric function do? Ie: $\sin^{-1}, \cos^{-1}$, or $\tan^{-1}$. What is the purpose of these inverse functions?

   Find the angles $\sin^{-1}\left(\frac{a}{b}\right) = \sin\theta$

   60 as example, $\sin^{-1}\frac{\sqrt{3}}{2} = \sin 60^\circ$

7. What does SOHCAHTOA stand for?

   $\text{Sin} = \frac{\text{opposite}}{\text{Hypotenuse}}$ $\text{Cos} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\text{Tan} = \frac{\text{opposite}}{\text{Adjacent}}$

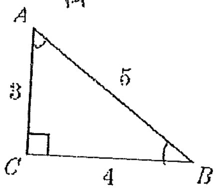
8. When I take sine 45 and divide it by cosine 45, does it equal to tangent 45? Why is it equal? Does sine an angle divided by cosine an angle always to tangent the angle? Why or why not?

   $$\frac{\frac{O}{H}}{\frac{A}{H}}$$

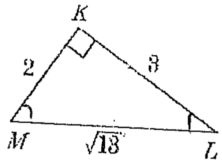
   $\frac{O}{H} \times \frac{H}{A} = \frac{O}{A}$, $\text{Tan} = \frac{O}{A}$

9. Find the ratios of the following functions and then solve for the angle:

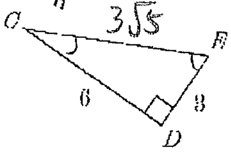
a) $\sin A = \frac{4}{5}$ $\frac{O}{H}$ $\cos B = \frac{4}{5}$ $\frac{A}{H}$



b) $\cos M = \frac{2\sqrt{13}}{13}$ $\frac{A}{H}$ $\tan L = \frac{2}{3}$ $\frac{O}{A}$

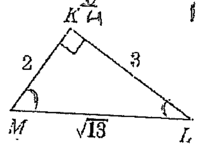


c) $\tan E = 2$ $\frac{O}{A}$ $\sin C = \frac{3}{3\sqrt{5}}$ $\frac{O}{H}$ $\frac{1}{\sqrt{5}}$ $\frac{\sqrt{5}}{5}$

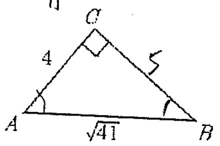


9+36

d) $\sin L = \frac{2\sqrt{13}}{13}$ $\frac{O}{H}$ $\cos M = \frac{2\sqrt{13}}{13}$ $\frac{A}{H}$



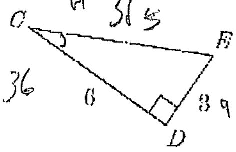
e) $\tan A = \frac{5}{4}$ $\frac{O}{A}$ $\cos B = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$ $\frac{A}{H}$



41-16 = 25

f) $\cos C = \frac{6}{3\sqrt{5}} = \frac{2\sqrt{5}}{5}$ $\frac{A}{H}$ $\sin C = \frac{\sqrt{5}}{5}$ $\frac{O}{H}$

$3\sqrt{5}$



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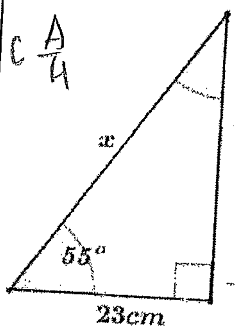
10. Find the length of the missing sides for each of the following triangles:

a) x = ?

C $\frac{A}{H}$

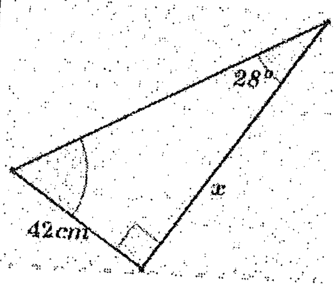


$\cos 55° = \frac{23}{x}$

$23 \div \cos 55$

$x = 40.099$

b) x = ?



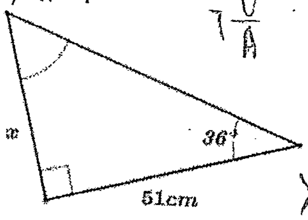
T $\frac{O}{A}$

$\tan 28° = \frac{42}{x}$

$42 \div \tan 28°$

$= 78.99$

c) x = ?



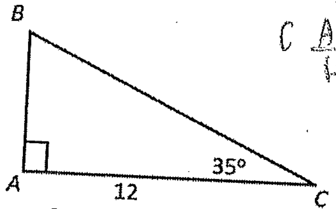
T $\frac{O}{A}$

$\tan 36 = \frac{x}{51}$

$51 \tan 36$

$x = 37.05$

d) AB = ?



T $\frac{O}{A}$ $\tan 35 = \frac{AB}{12}$

$12 \tan 35$

$= 8.4$

BC = ?

C $\frac{A}{H}$

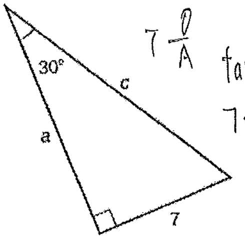
$\cos 35° = \frac{12}{BC}$

$12 \div \cos 35°$

$= 14.649$

e) a = ?

c = ?



T $\frac{O}{A}$

$\tan 30 = \frac{7}{a}$

$7 \div \tan 30$

$a = 12.12$

S $\frac{O}{H}$

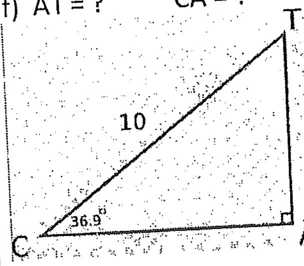
$\sin 30 = \frac{7}{c}$

$7 \div \sin 30$

$c = 14$

f) AT = ?

CA = ?



S $\frac{O}{H}$

$\sin 36.9 = \frac{AT}{10}$

$10 \sin 36.9$

$= 6.004$

C $\frac{A}{H}$

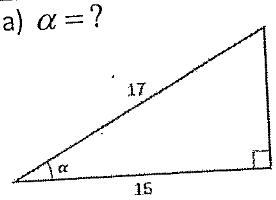
$\cos 36.9 = \frac{CA}{10}$

$10 \cos 36.9$

$= 7.99684$

11. Find the degree of the missing angle accurate to 3 decimal places:

a) $\alpha = ?$

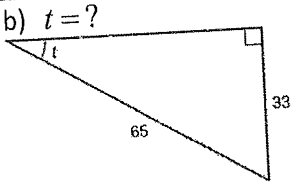


C $\frac{A}{H}$

$\cos^{-1} = \frac{15}{17}$

$\alpha = 28.07$

b) $t = ?$



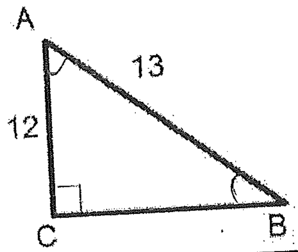
S $\frac{O}{H}$

$\sin^{-1} \frac{33}{65}$

$= 30.51$

c) $\angle B = ?$

$\angle A = ?$



S $\frac{O}{H}$

$\sin^{-1} \frac{12}{13}$

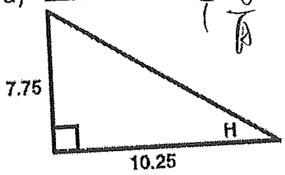
$B = 67.38°$

C $\frac{A}{H}$

$\cos^{-1} \frac{12}{13}$

$= 22.619864$

d) $\angle H = ?$

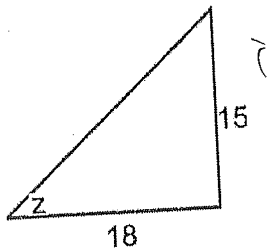


T $\frac{O}{A}$

$\tan^{-1} \frac{7.75}{10.25}$

$= 37.09°$

e) $\angle z = ?$

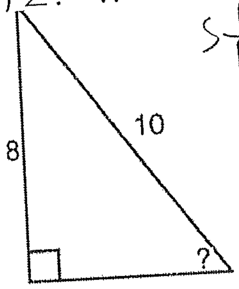


T $\frac{O}{A}$

$\tan^{-1} \frac{15}{18}$

$= 39.80°$

f) $\angle ? = ??$

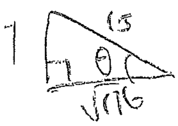


S $\frac{O}{H}$

$\sin^{-1} = \frac{8}{10}$

$= 53.13$

12. In a right triangle, you are told that $\sin\theta = \frac{7}{15}$. What is the value of $\cos\theta$ and $\tan\theta$?



$\cos\theta = \frac{A}{H} = \frac{\sqrt{176}}{15}$

$\cos\theta = \frac{4\sqrt{11}}{15}$

$\tan\theta = \frac{7\sqrt{11}}{44}$

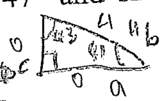
$\frac{\frac{O}{H}}{\frac{A}{H}}$ $= \frac{7}{15}$

$\frac{7}{\frac{15}{\frac{A}{15}}}$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

$= \frac{7}{A}$

13. If $\sin 43° = \frac{a}{b}$, then what are the values of $\cos 47°$ and $\sin 47°$ in terms of "a" and "b"?



$\sin 43° = \cos 47° = \frac{a}{b}$

$\sin(90° - 43°) = \sin 47° = \frac{\sqrt{b^2 - a^2}}{b}$

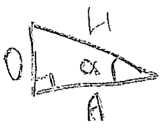
$\sin 43 = \frac{a}{b}$

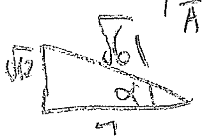
$\sin 47 = \frac{c}{b}$

$c = \sqrt{b^2 - a^2}$

14. Given that $0° < \alpha < 90°$, if $\tan\alpha = \frac{\sqrt{12}}{7}$ then what are the exact values of $\sin\alpha$ and $\cos\alpha$?

T $\frac{O}{A}$





$\sin\alpha = \frac{12}{61}$

S $\frac{O}{H}$

$\cos\alpha = \frac{7\sqrt{61}}{61}$

C $\frac{A}{H}$

15. Indicate if the following work shown below is correct? If there is a mistake, please indicate it:

a) $step1 := \frac{\sin 12°}{6}$ 0.034651948

$step2 := \sin 2°$ 0.0348994996

$step3 := 0.909297$

$\frac{\sin\theta(2)}{2} \neq \sin\theta$

b) $s1 := \cos 20° \times \sin 40°$

$s2 := 0.9 \times 0.64$

$s3 := 0.576$

✓

c) $s1:\ \cos\frac{3}{4} = \theta$

$s2:\ 0.9999 = \theta$

$\cos^{-1}\left(\frac{3}{4}\right)$

d) $s1: \tan(50°) = a$

$s2: -0.2719006 = a$

$\tan 50° = 1.191753593$

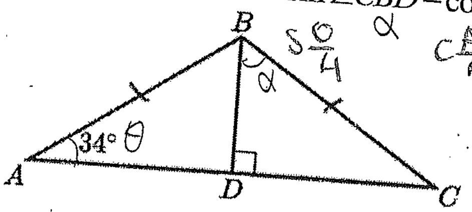
e) $s1: \sin(50+\theta) = \sin 40° + 0.8$

$s2: 50° + \theta = 40° + 0.8$

$s3: \theta = -9.2°$

sin does not cancel out

16. What is the value of $\sin\angle CBD - \cos\angle BAC$?



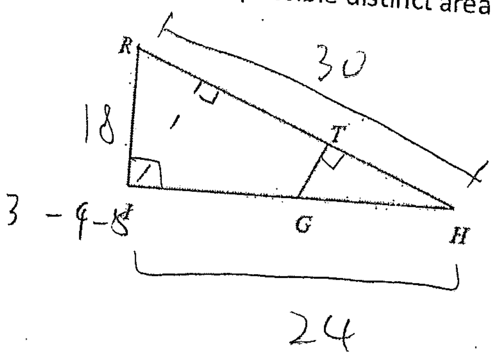
$\cos 34$

$\cos 34 = \frac{AD}{AB}$

$\sin CBD = \frac{CD}{BC}$

$\frac{CD}{BC} \cdot \frac{CD}{BC} = 0$

17. In the diagram, point "G" is on the line segment HI, point "T" lies on the line segment RH. $\angle RIG$ and $\angle HTG$ are both $90°$. RI=18, HI = 24 and the lengths of RT, HT, and GT are integers. Find the sum of all possible distinct area of quadrilateral IGTR.



3 - 4 - 5

6 - 8 - 10

9 - 12 - 15

12 - 16 - 20

$\frac{3\times4}{2}$

$\frac{6\times8}{2}$

$\frac{9\times12}{2}$

$\frac{16\times12}{2}$

$= \frac{12+48+108+192}{2}$

$= 180$

$\frac{18\times24}{2} \times 4 - 180$

$= 684$

Date: 12/1/2025

Name: Cody Lee

Math 10/11 Honors: Section 4.2 Radians and Angles in Standard Position

180 = 45×4 $\qquad \pi = 180°$

1. Convert the following angles to radians in terms of π. Show your work.

a) 60° $60° \times \frac{\pi}{180} = \frac{\pi}{3}$	b) 30° $30 \times \frac{\pi}{180}$ $\frac{\pi}{6}$	c) 150° $150 \times \frac{\pi}{180}$ $\frac{5\pi}{6}$	d) 210° $210 \times \frac{\pi}{180}$ $\frac{7\pi}{6}$
e) 90° $90 \times \frac{\pi}{180} = \frac{\pi}{2}$	f) 135° $135 \times \frac{\pi}{180}$ $\frac{3\pi}{4}$	g) 225° $225 \times \frac{\pi}{180}$ $\frac{5\pi}{4}$	h) 240° $240 \times \frac{\pi}{180}$ $\frac{4\pi}{3}$
i) 315° $315 \times \frac{\pi}{180} = \frac{7\pi}{4}$	j) 360° $360 \times \frac{\pi}{180}$ 2π	k) 330° $330 \times \frac{\pi}{180}$ $\frac{11\pi}{6}$	l) 1050° $1050 \times \frac{\pi}{180}$ $\frac{35\pi}{6}$

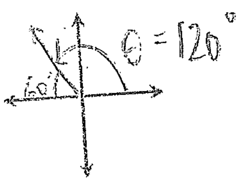
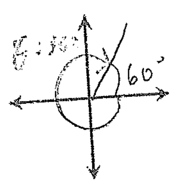
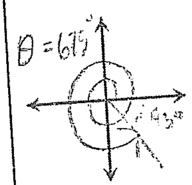
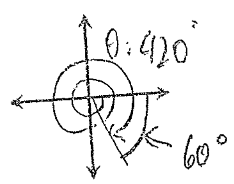
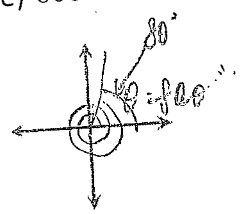
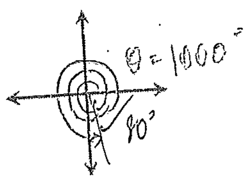
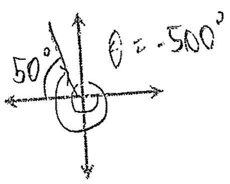
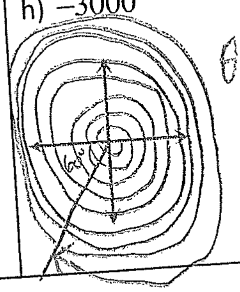
2. Convert the following to the nearest degree. Show your work.

a) $\frac{2\pi}{3}$ 120°	b) $\frac{2\pi}{6}$ 60°	c) $\frac{2\pi}{4}$ 90°	d) $-\frac{5\pi}{3}$ 300°
e) $\frac{16\pi}{12}$ 240°	f) $\frac{11\pi}{3}$ 660°	g) $\frac{7\pi}{6}$ 210°	h) $\frac{15\pi}{4}$ 675°
i) $\frac{\pi}{12}$ 15°	j) $\frac{5\pi}{6}$ 150°	k) $\frac{3\pi}{20}$ 27°	l) $\frac{22\pi}{9}$ 440°

3. Determine the arc length that subtends each angle at the center of the circle with radius 10cm.

a) 60° $= \frac{\pi}{3}(10)$ $\frac{10}{3}\pi$	b) 150° $= \frac{5\pi}{6}(10)$ $\frac{50\pi}{6} = \frac{25}{3}\pi$	c) 240° $\frac{40}{3}\pi$
d) $\frac{\pi}{12}$ $\frac{10}{12}\pi$ $\frac{5}{6}\pi$ //	e) $\frac{5\pi}{3}$ $\frac{50}{3}\pi$ //	f) $\frac{7\pi}{6}$ $\frac{70}{6}\pi = \frac{35}{3}\pi$ //

4. Graph each angle in standard position. Find the reference angle.

a) $\frac{2\pi}{3}$ 120 	b) $-\frac{10\pi}{6}$ $\frac{5}{3}\pi$ 	c) $\frac{15\pi}{4}$ = 675° 	d) $-\frac{7\pi}{3}$ 420° 
e) 800° 	f) 1000° 	g) –500° 	h) –3000°  θ = -3000

5. In what quadrants are the following angles in?

a) $35°$ Q1	b) $900°$ x-axis	c) $\frac{5\pi}{4}$ $225°$ Q3	d) $\frac{22\pi}{3}$ $= \frac{4}{3}\pi = 240°$ Q3
e) $-475°$ Q3	f) $-2590°$ Q4	g) $\frac{-9\pi}{5}$ Q1	h) $\frac{-17\pi}{4}$ $-\frac{1}{4}\pi$ Q4

6. Evaluate each of the following trigonometric functions without a calculator:

a) $\sin 60°$ $\frac{\sqrt{3}}{2}$	b) $\cos 90°$ 0	c) $\tan\frac{\pi}{2}$ undefined	d) $\cos\frac{\pi}{3}$ $60°$ $\frac{1}{2}$
e) $\sin 45°$ $\frac{\sqrt{2}}{2}$	f) $\tan 30°$ $\frac{\sqrt{3}}{3}$	g) $\sin\frac{\pi}{6}$ $\sin 30$ $\frac{1}{2}$	h) $\tan 0°$ 0

7. What is the smallest positive coterminal angle of $2000°$?

$2000 - 360 \times 30$

$200°$

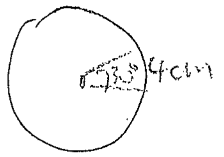
8. Give a general formula for all the coterminal angles of $-5200°$

$200° \pm (360°)n$; $n = 1, 2, 3 \ldots$

$\frac{-5200}{180} = -260$ -7

$= -160 = 200°$

9. Find the radius of a circle if an arc of 4cm subtends an angle of $30°$ on the circle.



$\frac{30}{360} = \frac{4}{2\pi r}$ $\frac{\pi}{6}$

$\frac{4}{12} = \frac{4}{2\pi r}$ $\frac{24}{\pi}$//

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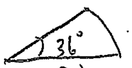
10. What is the length of an arc subtended from the sector angle $\pi/5$ if the circle has a radius of 20cm?



$\frac{\pi}{10\pi} = \frac{a}{2\pi r} = \frac{36°}{360°} = \frac{1}{10}$

$36°$

$\frac{1}{10} = \frac{a}{2\pi \times 20} = \frac{a}{40\pi}$ $a = 4\pi$//

11. What is the length of the radius of a circle with an arc length of 13.1 cm subtended from a sector of $42°$?

$\frac{42°}{360} = \frac{13.1}{2\pi r}$

$a = 13.1$

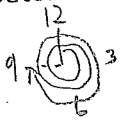
$\theta = 42°$ $\frac{7}{60} = \frac{13.1}{2\pi r}$

$786 = 14\pi r$

$\frac{786}{14\pi} = \frac{393}{7}\pi$//

$\doteq 17.87$ cm //

12. As the time changes from 1:00pm to 3:45pm on a clock, determine the change in radians of the minute hand. Determine the change in radians for the hour hand.



$360 \times 2 + 270$ $\quad 990°$ minute $\quad 990° \times \frac{\pi}{180°}$

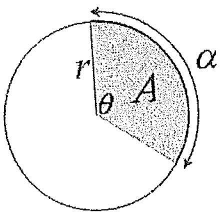
$\frac{11\pi}{2}$ minute hand //

$82.5° \times \frac{\pi}{180°} = \frac{11\pi}{24}$ hour hand //

$112.5° - 30°$ $\quad 82.5°$ hour

13. Derive a formula for the area, A, of a sector of a circle with radius "r", formed by an angle of θ radians. Derive a similar formula when the measure of the angle is in degrees

$A = \pi r^2 \times \frac{\theta_{DEG}}{360°}$



$\frac{\theta_{Deg}}{360°} = \frac{\theta_{RAD}}{2\pi} = \frac{a}{2\pi R} = \frac{A}{\pi R^2}$

14. If arc "a" is 6π cm long and the central $\theta = 72°$ then what is the area of the sector "A"?

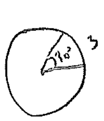
$\frac{6\pi}{2\pi R} = \frac{72°}{360°}$ $\quad \frac{1}{5} = \frac{6}{2r}$ $\quad \frac{A}{\pi(15)^2} = \frac{1}{5}$

$r = 15$

$5A = 15^2\pi$

$A = 45\pi$ // ≈ 141.37

15. Find the radius of a circle if an arc of 3 subtends an angle of $30°$ on the circle



$\frac{30°}{360°} = \frac{3}{2\pi r}$ $\quad \overset{18}{36} = 2\pi r$

$\frac{1}{12} = \frac{3}{2\pi r}$ $\quad \frac{18}{\pi} = r$ //

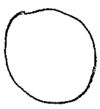
16. Find the angle in degrees if an arc length of 5cm has a radius of 6cm.

$\frac{5}{2\pi(6)} = \frac{\theta_{Deg}}{360°}$ $\quad \frac{150}{\pi} = \theta \approx 47.746°$ //

$\frac{5}{12\pi} = \frac{\theta}{360}$ $\quad \overset{150}{\cancel{1800}} = \cancel{12} \times \theta \times \pi$

17. When an object is moving in a circle, its "angular velocity" is the angle per unit time through which it rotates about the center. A car tire has diameter 64cm. Determine its angular velocity, in radians per second, when the car is travelling at 100km/h.

Deg → Rad



$2\pi r = 64\pi$ $\quad 100 \times 1000 \times 100$

$r = 32$ $\quad = 100,000,00$ cm $\div 64\pi$

≈ 49735.91972 ← rotate per hour

$\frac{312500}{3600} = 86.805$ Rad/secs // $\quad = \frac{1562500}{\pi}(2\pi) =$ Rad

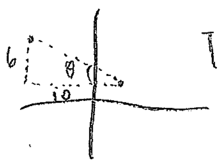
$= 312500$ Rad

$= \frac{3125}{72}$ rotated circle

$\frac{3125}{72} \times 180 = 7812.5°$

$7812.5 \times \frac{\pi}{}$

18. What is the smallest angle formed by the x-axis and the line through the points (2,1) and (-8,7)



T O/A $\tan^{-1} = \frac{6}{10} \approx 30.96°$ //

19. What is the sum of: $\sin^2(10°)+\sin^2(20°)+\sin^2(30°)+....+\sin^2(170°)$

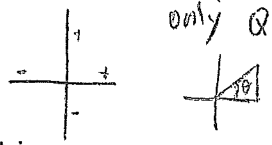
a) 1 b) 3 c) 5 d) 9 e) 10

$\sin(10°+170°) = 1$

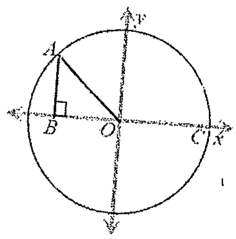
20. In the sequence below, each angle is in radians. What is the largest number of consecutive terms of this sequence that can be positive? x is in radians

$\cos x, \cos(x+1), \cos(x+2), \cos(x+3), \cos(x+4), \cos(x+5), \cos(x+6)$

$x \cdot \frac{180°}{\pi}$ only Q.1 & Q.3



21. Use a geometric approach with an unit circle to show that for any obtuse angle θ, $\sin\theta = \sin(\pi - \theta)$

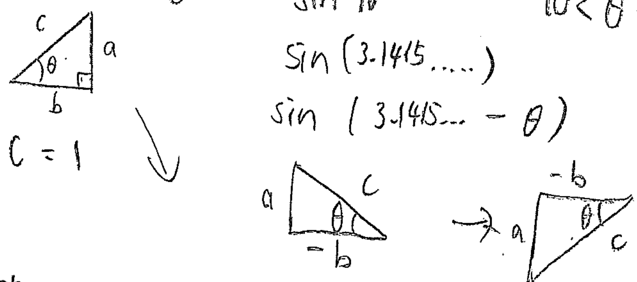


$\sin\theta = \frac{a}{c}$ $\sin\pi$ $90° < \theta < 180°$ 180

$\sin(3.1415.....)$

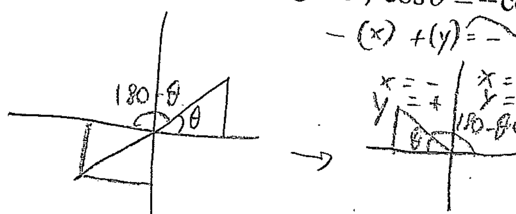
$\sin(3.1415... - \theta)$

$C = 1$



22. Use the same approach above to show that for any obtuse angle θ, $\cos\theta = -\cos(\pi - \theta)$

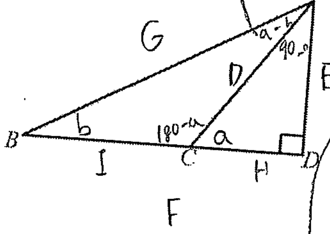
$-(x) + (y) = -$ 180



$+(x)$ $+(y) = +$

reference angle $= \theta$,

23. Challenge: Use the figure below to prove that $\sin(a-b) = \sin a\cos b - \sin b\cos a$. Hint: Use the "Sine Law" if necessary. $= a-b$ $90-b-(90-a)$ Sine law



$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$

$\frac{\sin(a-b)}{I} = \frac{\sin b}{D}$ $\frac{(I)\sin b}{D}$

$(D)\sin(a-b) = \frac{(I)\sin b}{D}$

$\frac{I}{D} \times \frac{E}{G} = \sin(a-b)$

$\frac{IE}{DG}$ ← $\sin(a-b)$

$\sin a \cos b - \sin b \cos a$

$= \frac{E}{D}$ $= \frac{I+H}{G}$ $\frac{E}{G}$ $= \frac{H}{D}$

$\frac{E}{D} \times \frac{I+H}{G} - \frac{E}{G} \times \frac{H}{D}$

$\frac{IE+EH}{GD} - \frac{EH}{GD}$

$= \frac{IE}{GD}$ ← $\sin a\cos b - \sin b\cos a$

$\therefore \sin(a-b) = \sin a\cos b - \sin b\cos a$ //

Date: 19/1/2025

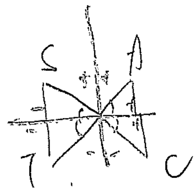
Name: Cody Lee

Math 10/11 Honors Section 4.3 Solving Angles in All Four Quadrants

1. If $\sin\theta$ is equal to a negative ratio, then which quadrants will the angle be? What if the ratio is positive, which quadrant is it in?

Negative: Q3, Q4

S θ A



Positive: Q1, Q3

2. If $\cos\theta$ is equal to a negative ratio, then which quadrants will the angle be? What if the ratio is positive, which quadrant is it in?

Q 2, Q3

Q1, Q4

3. If $\tan\theta$ is equal to a negative ratio, then which quadrants will the angle be? What if the ratio is positive, which quadrant is it in?

Q2, Q3

Q1, Q3

4. If θ is in quadrant 3, then which trig ratio will be negative? $\sin\theta$, $\cos\theta$, or $\tan\theta$?

$\sin\theta$ & $\cos\theta$

5. If θ is in quadrant 4, then which trig ratio will be negative? $\sin\theta$, $\cos\theta$, or $\tan\theta$?

$\sin\theta$ & $\tan\theta$

6. Solve for θ, with $0 \le \theta \le 360°$. [REMEMBER: There are TWO answers!]

a) $\sin\theta = 0.8$ θ = 53.13 233.13	b) $\cos\theta = 0.85$ θ = 31.788 211.788	c) $\tan\theta = 0.3$ θ = 16.699 196.699
a) $\sin\theta = -0.9$ θ = 115.84 295.84	b) $\cos\theta = 0.125$ θ = 82.819 262.819	c) $\tan\theta = 0.25$ θ = 14.036 194.036

60

g) $\cancel{3}\sin\theta+\cancel{2}=\cancel{0}$ $-\frac{5}{3}$

$-\frac{5}{3}$

undefined

h) $\tan^2\theta-5=0$

$(\tan\theta)^{\cancel{2}}=\pm\sqrt{5}$

$\theta=65.905$

245.905

114.094

294.094

i) $4\cos^2\theta-8=1$

$\frac{\sqrt{9}}{\sqrt{4}}=\pm\frac{3}{2}$

undefined

j) $(\cos\theta+1)(3\sin\theta-2)=0$

$\theta=180°$ $\theta=41.81$ $\frac{2}{3}$

$360°$ 221.81

k) $3\sin\theta=4\cos\theta$

$\tan\theta=\frac{4}{3}$

$\theta=53.13$

233.13

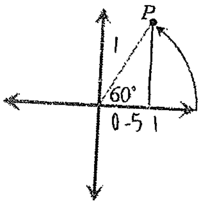
L) $\sin\theta=\cos\theta$

$\theta=45°$ $\tan^{-1}1$

$225°$

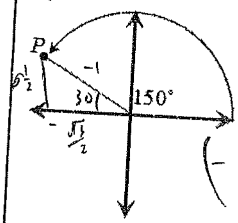
7. A point ``P`` created by the endpoint of a terminal arm is on the circumference of an unit circle of radius 1. Given the angle in standard position, find the coordinates of point `P`.

a) 60°



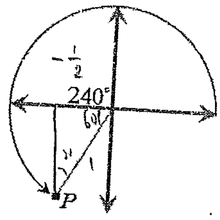
$\left(0.5,\ \frac{\sqrt{3}}{2}\right)$

b) 150°



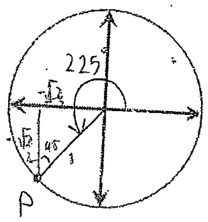
$\left(-\frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$

c) 240°



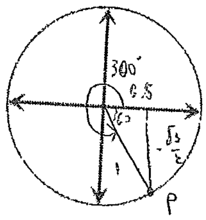
$\left(-\frac{1}{2},\ -\frac{\sqrt{3}}{2}\right)$

d) 225°



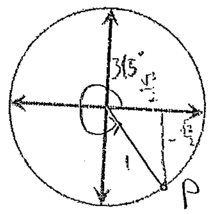
$\left(-\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2}\right)$

e) 300°



$\left(\frac{1}{2},\ -\frac{\sqrt{3}}{2}\right)$

f) 315°

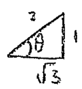


$\left(\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2}\right)$

8. Given each trig ratio, find the specified trig ratio without using a calculator:

a) $\sin\theta=0.5$

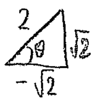
$\sin\theta=\frac{1}{2}$



$\cos\theta=\pm\frac{\sqrt{3}}{2}$

$\tan\theta=\pm\frac{1}{\sqrt{3}}$

b) $\cos\theta=\dfrac{-\sqrt{2}}{2}$

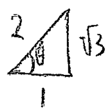


$\sin\theta=\ \pm\frac{\sqrt{2}}{2}$

$\tan\theta=\ \pm 1$

c) $\tan\theta=-\sqrt{3}$

$\frac{-\sqrt{3}}{1}$

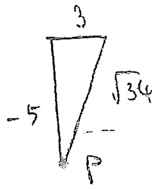


$\cos\theta=\ \pm\frac{1}{2}$

$\sin\theta=\ \pm\frac{\sqrt{3}}{2}$

d) $\sin\theta = 1/\sqrt{2}$ $\cos\theta = \pm \frac{1}{\sqrt{2}}$ $\tan\theta = \pm 1$	e) $\cos\theta = \frac{-\sqrt{3}}{2}$ $\sin\theta = \pm \frac{1}{2}$ $\tan\theta = \pm \frac{1}{\sqrt{3}}$	f) $\tan\theta = \frac{1}{\sqrt{3}}$ $\cos\theta = \pm \frac{\sqrt{3}}{2}$ $\sin\theta = \pm \frac{1}{2}$
g) $\sin\theta = -1$ $\frac{-1}{1}$ $\cos\theta = 0$ $\tan\theta =$ undefined	h) $\cos\theta = -0.5$ $-\frac{1}{2}$ $\sin\theta = \pm\frac{\sqrt{3}}{2}$ $\tan\theta = \pm\sqrt{3}$	i) $\tan\theta = -1$ $\frac{-1}{1}$ $\sin\theta = \pm\frac{1}{\sqrt{2}}$ $\cos\theta = \pm\frac{1}{\sqrt{2}}$

9. If the point P(3,-5) is on the terminal arm of an angle in standard position. What is the value of $\sin\theta \times \cos\theta$? Note: This point is not on the circumference of an unit circle.

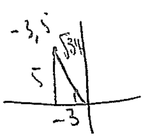


$\frac{3}{\sqrt{34}} \times \frac{-5}{\sqrt{34}} = \frac{-15}{34}$ //

10. Determine the following using exact value. No calculators:

a) $\cos\frac{5\pi}{4}$ $225^\circ = 45^\circ$ $-\frac{1}{\sqrt{2}}$	b) $\tan\frac{3\pi}{2}$ 90° undefined	c) $\sin\frac{7\pi}{6}$ 30° $-\frac{1}{2}$
d) $\tan\frac{-3\pi}{4}$ 135 225° $= 1$	e) $\cos\frac{11\pi}{6}$ 150° 330° $\frac{\sqrt{3}}{2}$	f) $\sin\frac{8\pi}{3}$ $480^\circ = 120^\circ$ $\frac{\sqrt{3}}{2}$
g) $\tan 3\pi = 180^\circ$ $= 0$	h) $\sin\frac{-5\pi}{3}$ $-300^\circ = 60^\circ$ $\frac{\sqrt{3}}{2}$	i) $\cos\frac{3\pi}{3}$ 180° -1

11. The point $(-3,5)$ is on the terminal arm of angle θ in standard position. Find the angle in radians to one decimal place.

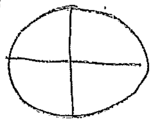


$\sin^{-1}\frac{5}{\sqrt{34}}$

$120.96 \div 180 \times \pi$

$= 2.111$ RAD //

12. A wheel with a radius of 1.5m is rotating at a angular velocity of 5.5radians per second. What is the speed of the wheel in m/s?



$2\pi r \times \frac{315.12}{360}$ $\quad 5.5 \times \frac{180}{\pi}$

$= 8.25$ m/s $\quad 315.12°$

13. When an object is moving in a circle, its angular velocity is the angle per unit time through which it rotates about the center. A car tire has diameter 64cm. Determine its angular velocity, in radians per second, when the car is travelling at 100km/h.

r = 32

$100 \times 1000 \times 100$

$= 1,000,000,0$ cm/h

$\frac{25000}{9}$ cm/secs

$\frac{25000}{9} \div 2\pi r = \frac{3125}{1236}\pi \times 2\pi$

$= 86.805$ RAD/sec //

14. Write an expression for the angular velocity, in radians per second, for a car tire with diameter "d" centimeters when the car is travelling at "x" kilometers per hour.

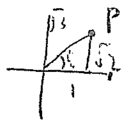
$\frac{X \cdot 100000}{3600} \div d\pi \times 2\pi$

$\frac{250}{9} \times 2\pi \div d\pi = \frac{250}{9} \times \frac{2}{d}$

$= \frac{500}{9d}$ RAD //

15. The angle θ is in the first quadrant and $\cos\theta = \frac{1}{\sqrt{3}}$. Draw a diagram to show the angle in standard position and a point "P" on its terminal arm. Determine the possible coordinates for "P"

$P\left(\frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right)$ //

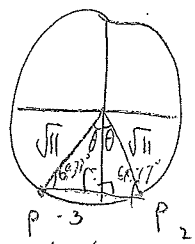


16. If $\sin\theta = -\frac{3}{\sqrt{11}}$, draw a diagram to show the angle(s) in standard position and a point "P" on its terminal arm. Determine the possible coordinates for "P" if it was on an unit circle.

$\theta = 115.23$
$\quad 295.23$

$\theta = 25.23°$

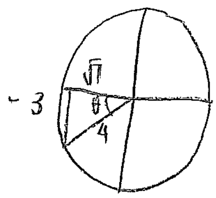
↳ $244.77°$ & $295.23°$



17. If $\tan\theta = -\frac{3}{\sqrt{7}}$, angle θ is in standard position, and its terminal arm is in quadrant II. What is the exact value of $\cos\theta$?

$228.59°$

$\cos\theta = \frac{\sqrt{7}}{4}$ //

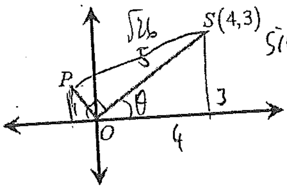


18. Point P(3,-5) is on the terminal arm of an angle in standard position. What is the value of $\sin\theta \times \cos\theta$?

refer to Q. 9

$\frac{-15}{34}$

19. In the figure, if $\overline{PO}$ is one unit long, find the exact coordinates of point "P".



$\sin 53.13° = \frac{1}{x}$

1.25

$\sin\theta = \frac{3}{5}$

$\sin 90-\theta = \frac{y}{1}$

$x = -0.6$

$y = 0.8$

$P(-0.6, 0.8)$

20. What is the value of $\sin\theta \times \tan\theta$ if point P(1.957, -0.412) is on a circle with a radius of 2units?

$\frac{\sin\theta}{\cos\theta} = \tan\theta \quad \cos\theta \rightarrow \frac{\sin\theta}{\cos\theta} \times \sin\theta =$

1.957, 2, -0.412

$\frac{O}{A} \times \frac{O}{H}$

$= 0.043368421$ //

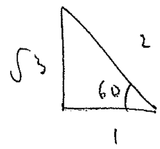
21. Evaluate: $\sin\left(\frac{\pi}{12}\right) - \cos\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right) - \cos\left(\frac{5\pi}{12}\right)$

15 30 45 60 75

$\sin 15 = \cos 75$

$\tan 45 - \sin 60 \times 2$

$1 \quad \sqrt{3}$



$\frac{\sqrt{3}}{2} \times 2$

$1-\sqrt{3}$ //

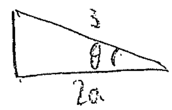
22. Find the exact value of $\sum_{k=1°}^{360°} \sin k$

$= 0$

$\sin 1 = \sin 181$

$\sin 180 = \sin 360$

23. If $\cos\theta = 2a/3$, then what is the value of $\tan\theta$ in terms "a"?



$\frac{\sqrt{9-4a^2}}{2a}$ //

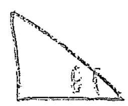
24. If $\cos x = 0$ and $\cos(x+k) = 0.5$, what is the smallest possible positive value of "k"?

$\cos 90 = 0$

$\cos 270 = 0$

$\cos 60 = 0.5$

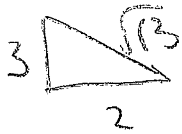
$\cos 240 = 0.5$



$k = 150$

25. If $K\cos\theta = 3$ and $K\sin\theta = 2$, where K>0, find the value of "K" k

$\cos\theta = \frac{3}{K} \frac{A}{H} \quad \sin\theta = \frac{2}{K} \quad \frac{O}{H}$



$9+4$

$k = \sqrt{13}$ //

26. If $A+B=180$, which of the following statements must be true?

I) $\sin A=\sin B$, ii) $\cos A=\cos B$ iii) $\tan A=-\tan B$

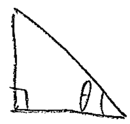
$\Downarrow$

$\cos A = -\cos B$

27. If $0\leq\theta\leq180^{\circ}$ and $\sin\theta\geq\cos\theta$, then what is the range of values for θ?

$45^{\circ}\leq\theta\leq180^{\circ}$

$\frac{O}{H} > \frac{A}{H}$



$\frac{O}{H} = \frac{A}{H}$, 45°

28. Which of the following is equal to $\cos\left(270^{\circ}-\theta\right)$? (No calculators)

I) $-\cos\theta$ ii) $\cos\theta$ iii) $-\sin\theta$ iv) $\sin\theta$

COS

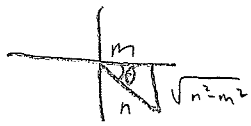
if $\theta = 90$

$A+B=180$, $\cos A = -\cos B$

$\cos 180 = -1$

$\sin 90 = 1$

29. Determine the exact value of $\sin\theta$ in terms of "m" and "n" if $\cos\theta=\frac{m}{n}$, where θ is in quadrant 4.



$\sin\theta = \frac{\sqrt{n^2-m^2}}{n}$ //

Lit studies 11
Math 12 H
Chem 11 H
Phy 11 AP 1
Bio 11 H
Comm sci 12
French 10
ocial ? AP mandarin ? AP econ ?

Date: 2/21/2025

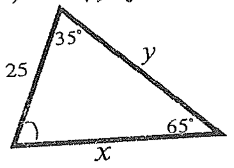
Name: Cody Lee

Math 10/11 Honors Section 4.4 Sine Law and Cosine Law

1. Indicate if you are to use the Sine Law or Cosine Law to find the missing side or angle. Then find the indicated side or angle. Show your work and steps:

2.

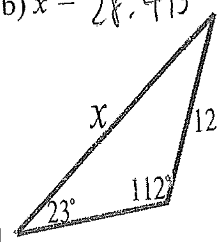
a) $x =$ 15.82 $y =$ 27.17



$\frac{\sin 65}{25} = \frac{\sin 35}{x}$

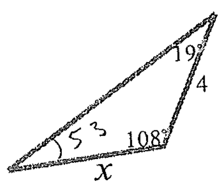
$\frac{\sin 65}{25} = \frac{\sin 80}{y}$

b) $x =$ 28.475



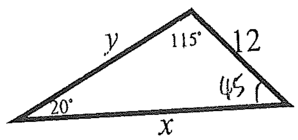
$\frac{\sin 23}{12} = \frac{\sin 112}{x}$

c) $x =$ 1.63



$\frac{\sin 53}{4} = \frac{\sin 19}{x}$

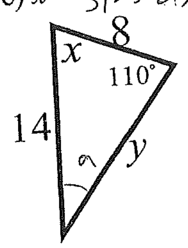
d) $x =$ 31.80 $y =$ 24.81



$\frac{\sin 115}{x} = \frac{\sin 20}{12}$

$\frac{\sin 20}{12} = \frac{\sin 45}{y}$

e) $x =$ 37.523 $y =$ 9.074

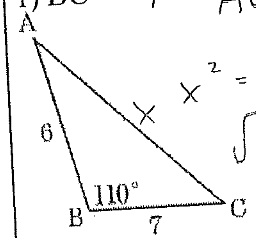


$\frac{\sin 110}{14} = \frac{\sin a}{8}$

$a = 37.523$

$\frac{\sin 110}{14} = \frac{\sin 37.532}{y}$

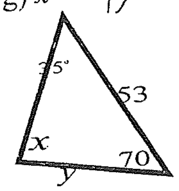
f) BC = 7 AC = 10.664



$x^2 = 6^2 + 7^2 - (12 \times 7)\cos 110$

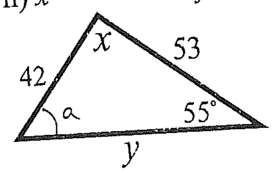
$\sqrt{36 + 49 - 84\cos 110}$

g) $x =$ 75° $y =$ 31.47



$\frac{\sin 75}{53} = \frac{\sin 35}{y}$

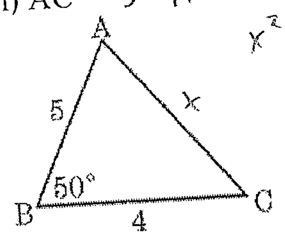
h) $x =$ $y =$



$\frac{\sin 55}{42} = \frac{\sin a}{53}$

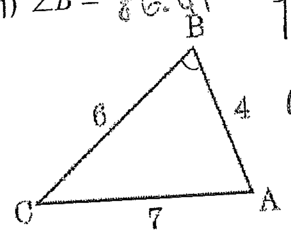
undefined //

i) AC = 3.91



$x^2 = 25 + 16 - 40\cos 50$

j) $\angle B =$ 86.41

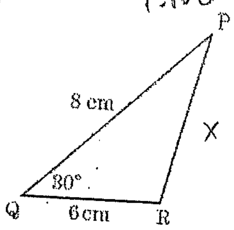


$7^2 = 6^2 + 4^2 - 48\cos B$

$49 = 36 + 16$

$48\cos B = 3$

k) PR = 4.106

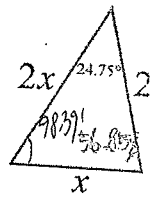


$x^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \cos 30$

64 36

100

L) x = ?



$x^2 = (2x)^2 + (2x+3)^2 - 2(2x+3)(2x)\cos 24.75$

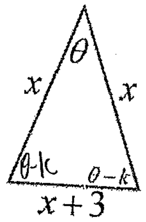
$4x^2 + 12x + 9 - (8x^2 + 12x)\cos 24.75$

$-0.265 \quad 1.102 \quad -7.265x^2 - 10.89$

$x = 8.266$ //

m)

$k = 31.6246°$ $x = ?$ $\theta = ?$



$\theta = 81.083°$

$3\theta - 2k = 180$

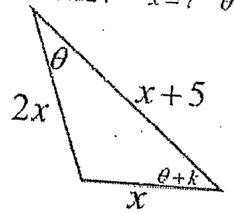
$(x+3)^2 = x^2 + x^2 - 2x^2 \cos 81.083$

$x^2 + 6x + 9 = x^2 + x^2 - 2x^2 \cos 81.083$

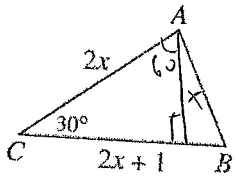
$x = 10.0$ //

n)

$k = 46.44324°$ $x = ?$ $\theta = ?$



3. In the diagram, $AC = 2x$, $BC = 2x+1$ and $\angle ACB = 30°$. If the area of $\triangle ABC$ is 18, what is the value of "x"?

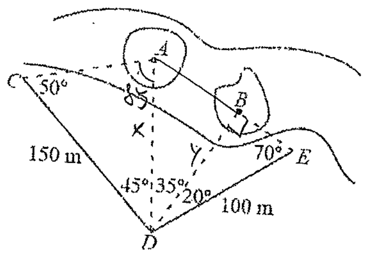


$\frac{(2x+1)x}{2} = 18$

$2x^2 + x - 36 = 0$

$x = 4$ //

4. In the diagram, points A and B are located on islands in a river full of rabid aquatic goats. Determine the distance from A to B, to the nearest meter.



$\frac{\sin 85}{150} = \frac{\sin 50}{x}$

$x = 115.35$

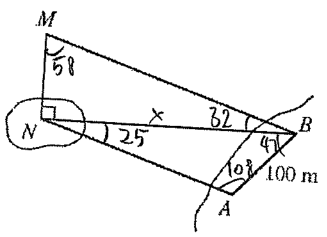
$\frac{\sin 90}{100} = \frac{\sin 70}{y}$

$y = 93.969$

$a^2 = 115.35^2 + 93.969^2 - 2 \times 115.35 \times 93.969 \cos 35$

66.16 //

5. In determining the height, MN, of a tower on an island, two points A and B, 100 meters apart, are chosen on the same horizontal plane as "N". If $\angle NAB = 108°$, $\angle ABN = 47°$, and $\angle MBN = 32°$, determine the height of the tower to the nearest meter.



$x = 225.03$

$\frac{\sin 25}{100} = \frac{\sin 108}{x}$

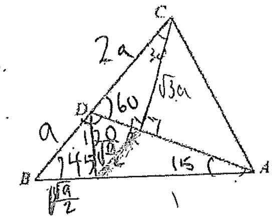
$\frac{\sin 58}{x} = \frac{\sin 32}{y}$

ans $\frac{\sin 32}{\sin 58}$

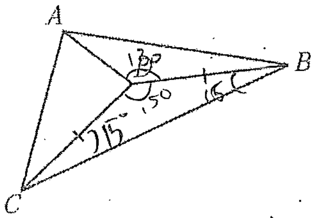
≈ 141 m //

6. In triangle ABC, $\angle ABC = 45°$. Point "D" is on $\overline{BC}$ so that $2\square BD = CD$ and $\angle DAB = 15°$. Find $\angle ACB$.

a) 54° b) 60° c) 72° d) 75° e) 90°

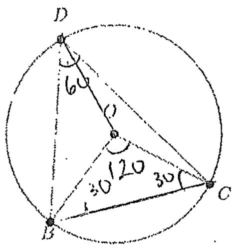


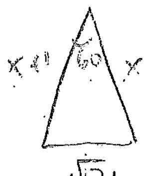
7. In the diagram, $DC = DB, \angle DCB = 15°$, and $\angle ADB = 130°$. What is the measure of $\angle ADC$?

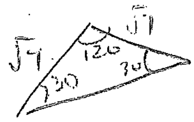


$ADC = 80°$ //

8. In the diagram, the circle has radius $\sqrt{7}$ and centre O. Points D, B, and C are on the circle. If $\angle BOC = 120°$ and $DC = DB + 1$, determine the length of DB.







$\frac{\sin 30}{\sqrt{7}} = \frac{\sin 120}{x}$

$\sqrt{21}$

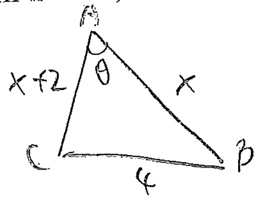
$\sqrt{21}^2 = x^2 + (x+1)^2 - 2x(x+1)\cos 60$

$2x^2 + 2x + 1 - 2x^2 + 2x$

$21 = x^2 + x + 1 - x^2 - x$

$\boxed{5 //}$

In $\square ABC, BC = 4, AB = x, AC = x + 2,$ and $\cos(\angle BAC) = \frac{x+8}{2x+4}$. Determine all possible values of "x".



$4^2 = x^2 + (x+2)^2 - 2x(x+2)\frac{x+8}{2x+4}$

$16 = 2x^2 + 4x + 4$

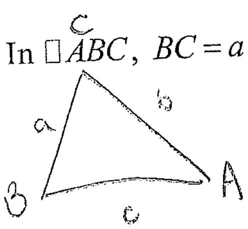
-12

$x^2 + 8x$

$x^2 - 4x - 12$

6 //

9. In $\square ABC$, $BC = a$, $AC = b$, $AB = c$, and $a < \frac{1}{2}(b + c)$. Prove that $\angle BAC < \frac{1}{2}(\angle ABC + \angle ACB)$



$\angle A \quad \angle B + \angle C$

$a < \frac{1}{2}(b+c)$

$\angle A < \frac{1}{2}(180 - \angle A)$

$\angle A = \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $0 < \angle A < 60°$

$\sin A = \frac{a \sin B}{b}$

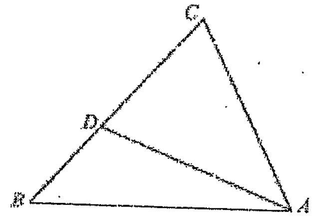
$2a < (b+c)$

$-\frac{1}{2} \leq \cos\theta \leq 1$

10. In triangle ABC, $\angle ABC = 45°$. Point "D" is on BC so that $2\square BD = CD$ and $\angle DAB = 15°$. Find $\angle ACB$

a) 54° b) 60° c) 72° d) 75° e) 90°

(Q. 6)

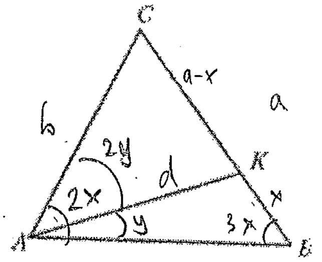


10. Challenge: In the diagram, $2\angle BAC = 3\angle ABC$ and "K" lies on BC such that $\angle KAC = 2\angle KAB$. Suppose that $BC = a$, $AC = b$, $AB = c$, $AK = d$, and $BK = x$ a) Prove that $d = \frac{bc}{a}$ and $x = \frac{a^2 - b^2}{a}$

$\frac{(a+b)(a-b)}{a}$

$\frac{\sin 3x}{b} = \frac{\sin 2x}{a}$

$\frac{a}{b} = \frac{2}{3}$



$\frac{\sin 3x}{d} = \frac{\sin y}{x}$

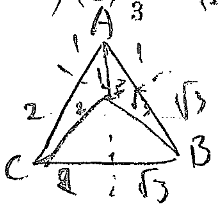
b) Prove that $(a^2 - b^2)(a^2 - b^2 + ac) = b^2c^2$

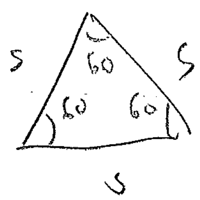
$(a+b)(a-b) \cdot (a^2 + ac - b^2)$

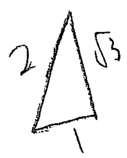
11. Challenge:

Suppose that $\triangle ABC$ is an equilateral triangle of side length s, with the property that there is a unique point P inside the triangle such that $AP = 1$, $BP = \sqrt{3}$, and $CP = 2$. What is s?

(A) $1 + \sqrt{2}$ (B) $\sqrt{7}$ (C) $\frac{8}{3}$ (D) $\sqrt{5 + \sqrt{5}}$ (E) $2\sqrt{2}$







$x + 2x = x + \sqrt{3}x = 2x + \sqrt{3}x$

S A
T C

Name: Cody Lee

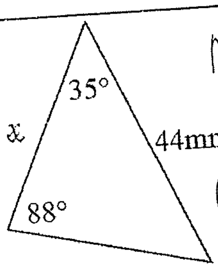
Date: 23/12/2025

Math 10/11 Honours Section 4.5 Ambiguous Case of Sine Law

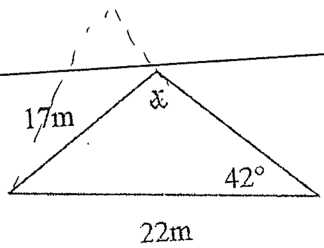
1. Given each equation, solve for all values of θ where $0 \le \theta \le 360^\circ$. Note: There are two angles!!

a) $\sin\theta = \frac{2}{3}$ $\theta_1 = $ 41.81 $\theta_2 = $ 138.19	b) $\sin\theta = \frac{4}{5}$ $\theta_1 = $ 53.13 $\theta_2 = $ 126.87	c) $\sin\theta = -0.55$ $\theta_1 = $ 326.63 $\theta_2 = $ 213.37
d) $\sin\theta = \frac{-\sqrt{2}}{2}$ $\theta_1 = $ 315 $\theta_2 = $ 225	e) $\sin\theta = \frac{-\sqrt{3}}{2}$ $\theta_1 = $ 300 $\theta_2 = $ 240	f) $\sin\theta = \frac{4}{\sqrt{7}}$ > 1 undefined $\theta_1 = $ ____ $\theta_2 = $ ____

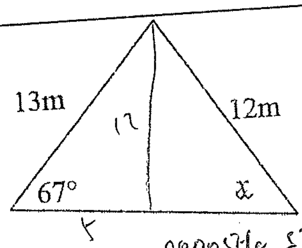
2. Given each of the following triangles, indicate whether if there would be an ambiguous case. State the reason why or why not:



not ambiguous (given 2 angles)

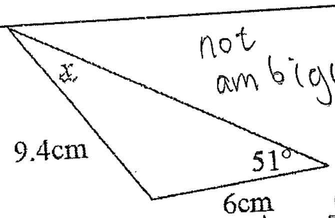


ambiguous (not given the remaining side & the angle (opposite) of the remain side)



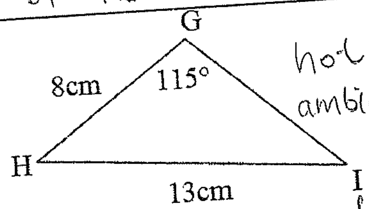
ambiguo

opposite side of giv angle is smaller than the other given side



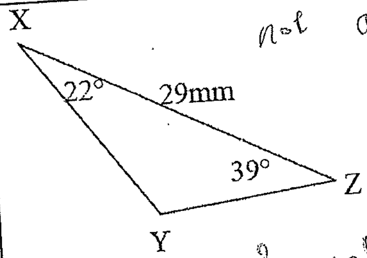
not ambiguous

opposite side of given angle is bigger



not ambiguous

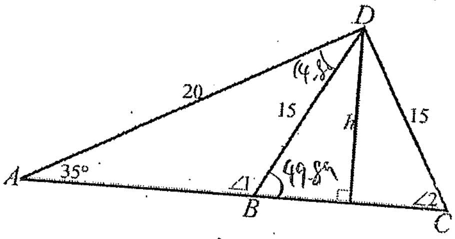
opposite side of given angle is bigger



not ambi

given 2 angles

Find the value of $\angle 1, \angle 2, h, BC, and\ AB$



$\frac{20 \sin 35}{15} = \frac{\sin \angle 1}{20}$

$\angle 1 = \cancel{49.89}$

or 130.11

$h^2 + \left(\frac{BC}{2}\right)^2 = 15^2$

225

$BC = 19.32$

$\frac{\sin 90}{\cancel{15}} = \frac{15 \sin 49.89}{h}$

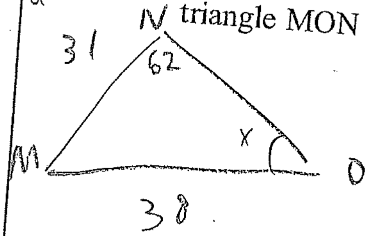
$h = 11.47$

$AB : 15 \quad \frac{\sin 14.89}{AB} = \frac{\sin 35}{\cancel{15}}$

$AB = 6.72$

$\angle 1 =$ 130.11 $\angle 2 =$ 49.89 $h =$ 11.47 $BC =$ 19.32 $AB =$ 6.72

not ambiguous 4. Given that length MN is 31m, MO is 38m and angle MNO is 62degrees. Draw a diagram for triangle MON and then find the value of $\angle MON, \angle OMN, and\ \overline{ON}$



$\frac{\sin 62}{38} = \frac{\sin x}{31}$

$x = 46.08$

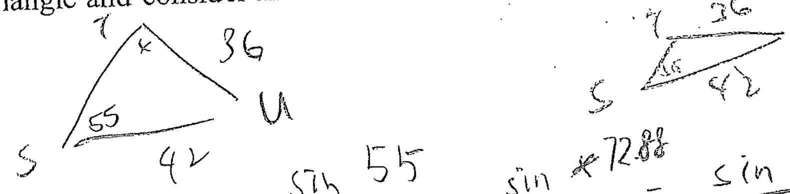
$\frac{\sin 62}{38} = \frac{\sin 71.92}{x}$

$\angle MON =$ 46.08 (ACUTE) $\angle OMN =$ 71.92 $ON =$ 40.91

not ambiguous

$\angle MON =$ ______ (OBTUSE) $\angle OMN =$ ______ $ON =$ ______

e) Triangle STU has side TU = 36, US = 42, and angle UST = 55degrees. Draw a diagram for the triangle and consider all cases. Find the length and angles below:



$\frac{\sin 55}{36} = \frac{\sin \cancel{x} 72.88}{42} = \frac{\sin 52.12}{Y}$

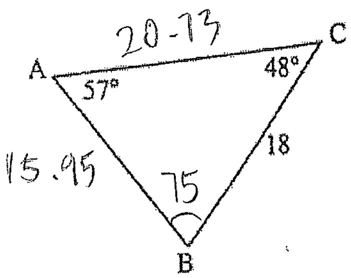
$x = 72.88°$

$\frac{\sin 55}{36} = \frac{\sin 17.88}{x}$

$\angle STU =$ 72.88° (ACUTE) $\angle T =$ 52.12° $ST =$ 34.69

$\angle STU =$ 107.12 (OBTUSE) $\angle T =$ 17.88° $ST =$ 13.49

5.. Find the area of the following triangle.

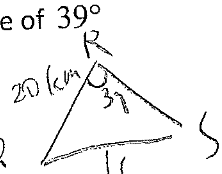


$\frac{\sin 57}{18} = \frac{\sin 75}{X} = \frac{\sin 48}{y}$

138.65 //

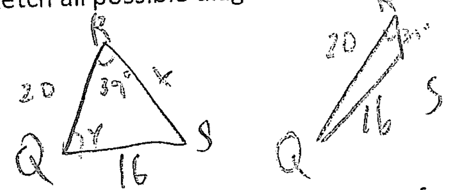
6. A lighthouse at point Q is 20 km from a yacht at point R and 16 km from a sailboat at point S. From the yacht, the lighthouse and the sailboat are separated by an angle of 39°

a) Is it necessary to consider the ambiguous case? Explain.



Yes, the opposite of given angle 16 is smaller than the other side

b) Sketch all possible diagrams for this situation.



$\frac{\sin 39}{16} = \frac{\sin \cancel{x} 12.87}{\cancel{20}}$ x 12.87°

128.12

c) Determine all possible the distances from the yacht to the sailboat, to the nearest tenth of a kilometre.

$\frac{\sin 39}{16} = \frac{\sin 89.12}{\cancel{20}\ X}$ $Y = 89.12°$

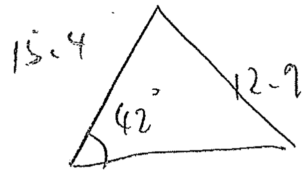
$X = 25.42$ or 5.66

X = 25 km or 6 km //

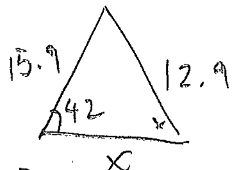
balloon into an active part of the cloud. Jason's rope is 15.4 m long and makes an angle of $42°$ with the ground. Belle's rope is 12.9 m long.

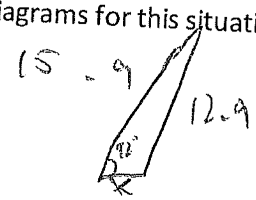
a) Is it necessary to consider the ambiguous case? Explain.

yes, the opposite side of given angle is smaller than the other one



b) Sketch all possible diagrams for this situation.





$\frac{\sin 42}{12.9} = \frac{\sin x}{\cancel{15.9}}$ 13.56° x

4.52

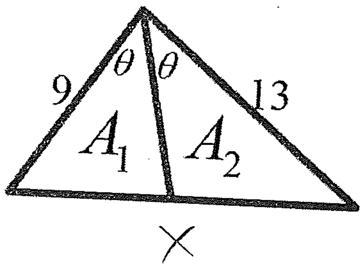
c) Determine all possible the distances between Jason and Sammy to the nearest tenth of a meter.

$\frac{\sin 42}{12.9} = \frac{\sin x}{\cancel{15.9}}$ 82.43 x

55.56° 19.11

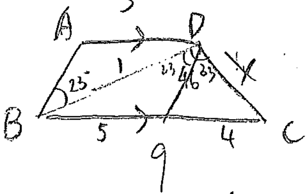
19 m or 5 m //

8. Given the following triangle with an area of 200units², what is the area of A_1 and A_2 ?



$\frac{A_1}{A_2} = \frac{9}{13}$ $\frac{900}{11}$ (A_1) $\frac{1300}{11}$ (A_2)

9. Trapezoid ABCD has $AD \parallel BC$, $BD = 1$, $\angle DBA = 23°$ and $\angle BDC = 46°$. The ratio of $BC : AD$ is $9:5$. What is the length of CD? (AMC12)



$\frac{4}{5}$ //

$\frac{1}{5} = \frac{x}{4}$

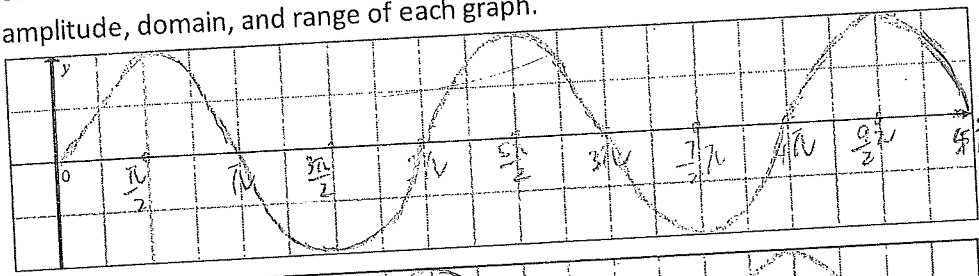
$\frac{4}{5} = x$

Date: 24/2/2025

name: Cody Lee

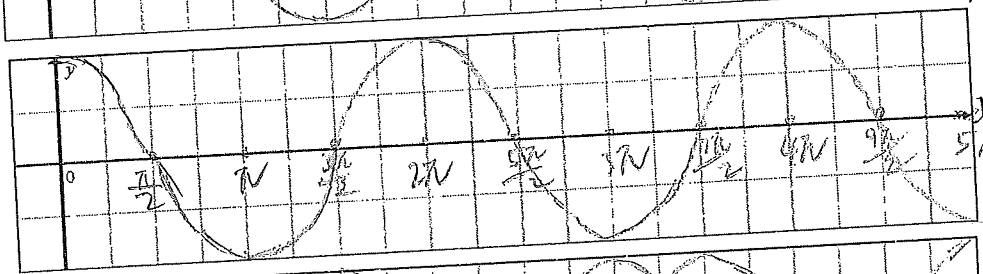
Section 4.6 Graphing Sine Cosine and Tangent Functions

1. Graph two periods of the Sine / Cosine / and Tangent function from $0 \le \theta \le 4\pi$. Label and set the increments on the X/Y-axis, write a general formula for all the x-intercepts, vertical asymptotes (if any). Indicate the period, amplitude, domain, and range of each graph.

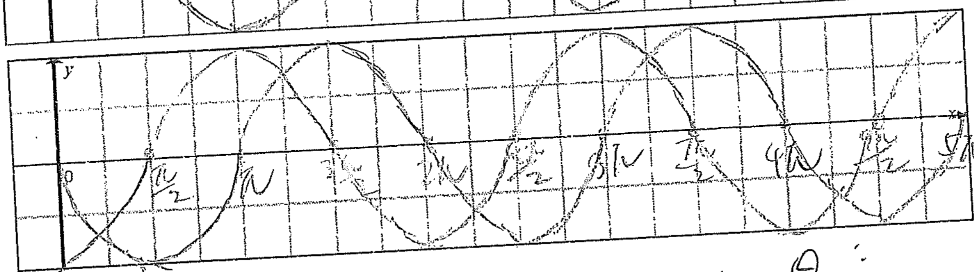


$y = \sin\theta$

period : 2π
amplitude : 1
x - int : $(0.5+n)\pi$, n is integer
D : $x \in R$
R : $-1 \le y \le$



$y = \cos\theta$



$y = -\sin\theta$

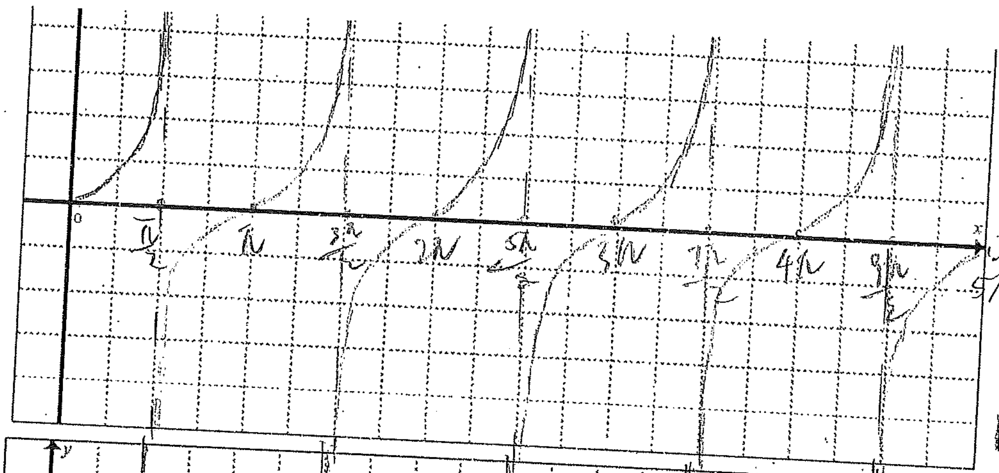
$y = -\cos\theta$

$-\cos\theta$ $-\sin\theta$

$\sin\theta$:
x - int : $(n)\pi$, n = integer
period : 2π
Amplitude = 1
D : $x \in R$
R : $-1 \le y \le 1$

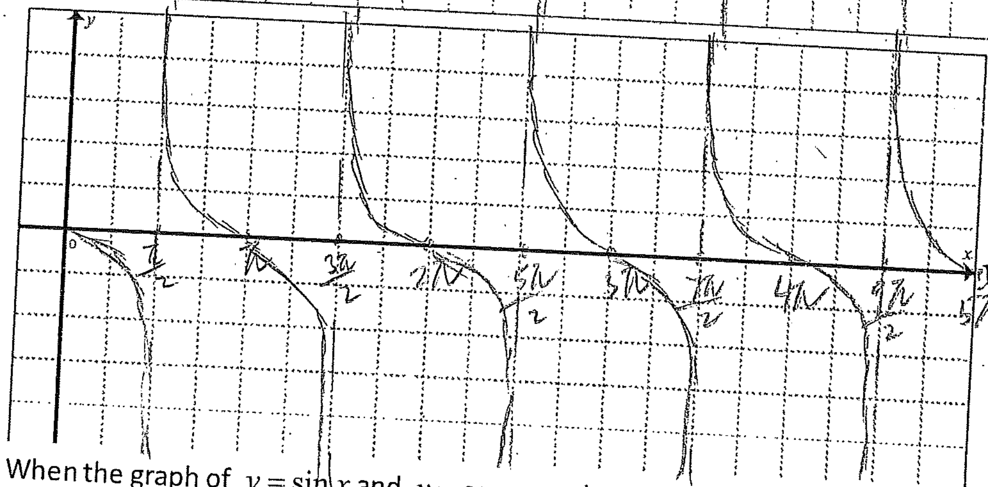
$-\cos\theta$:
x - int : $(n+0.5)\pi$
period : 2π
amplitude : 1
D : $x \in R$
R : $-1 \le y \le 1$

$-\sin\theta$:
x - int : $(n)\pi$
period : 2π
amplitude : 1
D : $x \in R$
R : $-1 \le y \le$

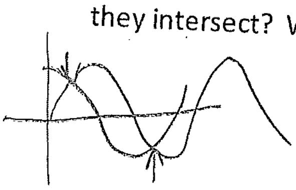


x - int : (n) π
period : π
amplitude : undefined
D: $x \in R$, $x \neq (n+0.5)\pi$
R: $y \in R$

Same



2. When the graph of $y = \sin x$ and $y = \cos x$ are drawn on the same graph for $0 < x < 2\pi$ in which quadrants do they intersect? What are the coordinates of the points of intersection?



45°, 225° → $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{5}{4}\pi, \frac{-\sqrt{2}}{2}\right)$

Q1, Q3

3. Given that $\sin\theta > 0$ and $\cos\theta < 0$, what is the range of possible values of θ if $0 < \theta < 2\pi$?

0 to 180°
90° to 270°

→ $90° < \theta < \pi$

4. Indicate TRUE or FALSE: $\sin\theta > 0$ and $\cos\theta > 0$, then $\tan\theta$ can be either positive or negative.

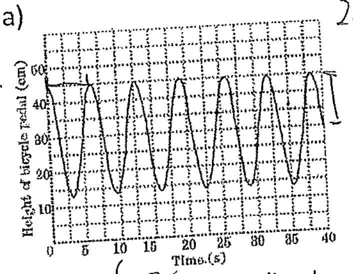
0 to 180°

False

$\frac{\sin}{\cos} = \tan$ $\frac{\text{positive}}{\text{positive}}$ can never = negative

5. Given each of the following trigonometric graphs, indicate the amplitude and period

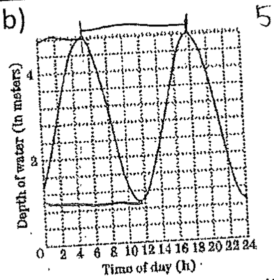
a)



2.5×2.5

Period= 6.25 Amplitude= 15

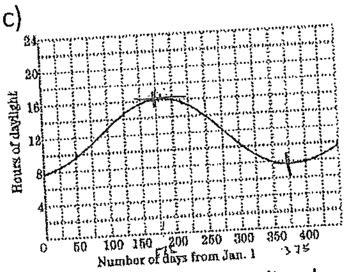
b)



$5 \rightarrow 18$

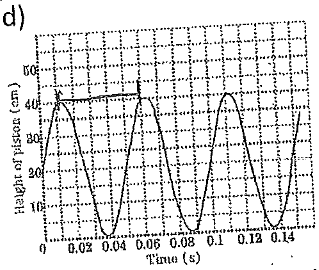
Period= 13 Amplitude= 2

c)



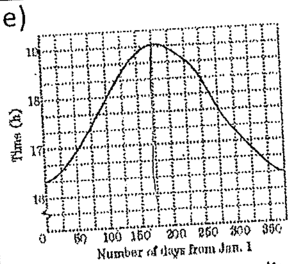
Period= 400 Amplitude= 4

d)



Period= 0.05 Amplitude= 20

e)



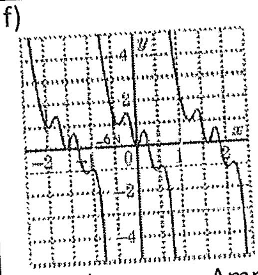
$19 - 16\frac{1}{3}$

$2\frac{2}{3}$

$1\frac{2}{3}$

Period= 375 Amplitude=

f)



$-2 \rightarrow -0.5$

Period= 1.5 Amplitude= undefined

6. How do the intersections of $y=\sin x$ and $y=\cos x$ relate with the graph of $y=\tan x$?

$\frac{\sin x}{\cos x} = \tan x$

when $\sin x$ & $\cos x$ intersect,

$\sin x = \cos x$

$\hookrightarrow \frac{\sin x}{\cos x} = 1$, $\tan x = 1$

7. How many units should the graph of $y=\sin x$ be shifted horizontally so that it will overlap the graph of $y=\cos x$?

$\frac{\pi}{2}$ units to left

8. When the graph of $y=\sin x$ and $y=0.5$ are drawn on the same graph for $0<x<2\pi$ in which quadrants do they intersect? What are the coordinates of the points of intersection?

$\left(\frac{\pi}{6}, 0.5\right)$ Q1 $\left(\frac{5\pi}{6}, 0.5\right)$ Q2

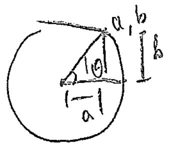
9. What is the amplitude and period of the graph $y=A\sin(Bx)$ if $A=-3$ and $B=2$?

$-3 \sin(2x)$

period $= \pi$

amplitude : 3

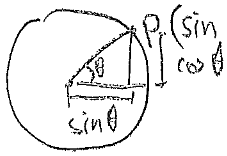
10. Given that the terminal arm intersects the unit circle at coordinates (a,b), what is the reference angle and the angle in standard position?



$\tan\theta = \frac{b}{a}$

ref : $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

11. If point "P" is on the unit circle with coordinates defined by $(\sin\theta, \cos\theta)$, what is θ in standard position?

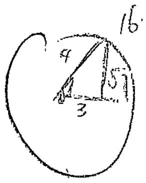


$\tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$ //

$\frac{A}{H} \div \frac{O}{H} = \frac{A}{O}$

12. Given the identity $\sin 2a = 2\sin a \times \cos a$, what is the value of $\sin 2d$ if $\cos d = \frac{3}{4}$ and "d" is in quadrant 1?

Find the exact value.



$16-9$

$\frac{\sqrt{7}}{4} \times \frac{3}{4} \times \frac{2}{1} = 2\cos d \times \sin d$

$\frac{3\sqrt{7}}{8}$ //

13. If $\cos\theta = \frac{a^2-b^2}{a^2+b^2}$ and $0° \le \theta \le 90°$, find the value of $\sin\theta$:

$\frac{A}{H}$ $x-y$ $x+y$

$\sqrt{(x+y)^2-(x-y)^2}$

$(x+y+x-y)(x+y-x+y)$

$(2x)(2y)$

$\sqrt{4a^2b^2}$

a) $\frac{2ab}{a^2+b^2}$ b) $\frac{4ab}{a^2+b^2}$ c) $\frac{2a^2b^2}{a^2+b^2}$ d) $\frac{4a^2b^2}{a^2+b^2}$ e) $\frac{a^2b^2}{2a^2+2b^2}$

14. If $0° \le \theta \le 180°$ and $\sin\theta \ge \cos\theta$, then:

a) $0 \le \theta \le 45°$ b) $45° \le \theta \le 90°$ c) $45° \le \theta \le 180°$ d) $90° \le \theta \le 180°$ e) $0 \le \theta \le 90°$

$\sin\theta = \cos\theta$

$45°$, $225°$

$\sin\theta > \cos\theta$

15. $\cos(270° - \theta) =$

a) $-\cos\theta$ b) $\cos\theta$ c) $-\sin\theta$ d) $\sin\theta$ e) $\sin\theta\cos\theta$

16. If $\sin 2a < 0$, $\cos a - \sin a < 0$, which quadrant is angle a in?

a) I b) II c) III d) IV

$a > 90$

~~Q1~~

Q2	~~Q1~~
~~Q3~~	~~Q4~~

17. In ΔABC, $2\cos B \cos A = \sin C$. What kind of shape is the triangle?

a) Right triangle b) Equilateral triangle c) 45-45-90 triangle d) Isosceles triangle

18. $0 < \beta < 2\pi$ what does β need to be in order for $\sin\beta > \cos\beta$ to be true?

A. $\frac{\pi}{4} < \beta < \frac{\pi}{2}$ and $\pi < \beta < \frac{5}{4}\pi$

B. $\frac{\pi}{4} < \beta < \pi$

C. $\frac{\pi}{4} < \beta < \frac{5}{4}\pi$

D. $\frac{\pi}{4} < \beta < \pi$ *and* $\frac{5}{4}\pi < \beta < \frac{3}{2}\pi$

$45 \le \theta \le 180$

19. Angle A, B are both acute angles. Point P has coordinates (cosB-sinA, sinB-cosA) Which quadrant is point P in?

$90 > A, B$

a) I

b) II

c) III

d) IV

$(-, +)$

20. sinα>sinβ Which of the following is true?

a) If α,β are in the quadrant I, then cosα>cosβa

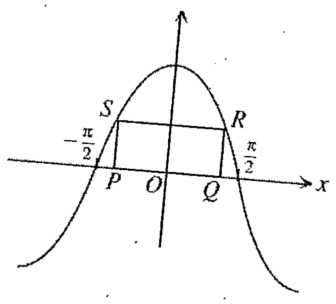
b) If α,β are in the quadrant II, then tanα>tanβ X

c) If α,β are in the quadrant III, then cosα>cosβ

d) If α,β are in the quadrant IV, then tanα>tanβ

sin a > sinb

21. A rectangle PQRS has side PQ on the x-axis and touches the graph of $y = k\cos(x)$ at the point "S" and "R" shown. If the length of PQ is $\frac{\pi}{3}$ and the area of the rectangle is $\frac{5\pi}{3}$, what is the value of "k"?


S
R
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
P
O
Q
x

Name: Cody Lee

Date: 4/3/2024

Math 10/11 Honors: Section 4.7 Cosecant Secant and Cotangent Functions

1. Determine each value to 3 decimal places: csc = sin sec = cos

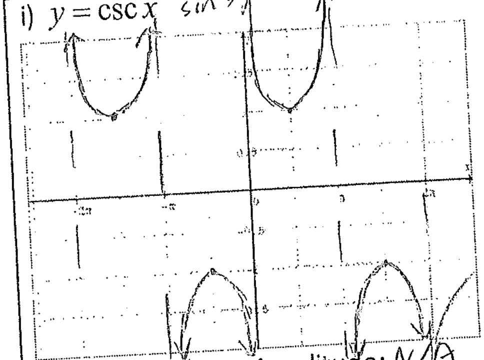
a) $\csc 110°$ 1.064	b) $\cot 130°$ −0.839	c) $\sec 95°$ −11.474	d) $\csc 64°$ 1.113
e) $\cot 233°$ 0.754	f) $\sec 100°$ −5.759	g) $\cot 45°$ 1	h) $\sec 112°$ −2.669

2. Determine the exact value of each of the following without a calculator

a) $\csc 45°$ $\frac{2}{\sqrt{2}}$	b) $\cot 180°$ undefined	c) $\sec 60°$ 2	d) $\csc 135°$ 1.414
e) $\cot\frac{\pi}{3}$ 0.577	f) $\sec\frac{2\pi}{3}$ −2	g) $\cot 225°$ 1	h) $\csc 300°$ −1.155
i) $\sec\frac{11\pi}{6}$ 1.155	j) $\cot\frac{4\pi}{3}$ 0.577	k) $\csc\pi$ undefined	L) $\sec\frac{5\pi}{6}$ −1.155

3. Graph the following function for $-2\pi \le \theta \le 2\pi$. Indicate the Period, Amplitude, Domain, and Range:

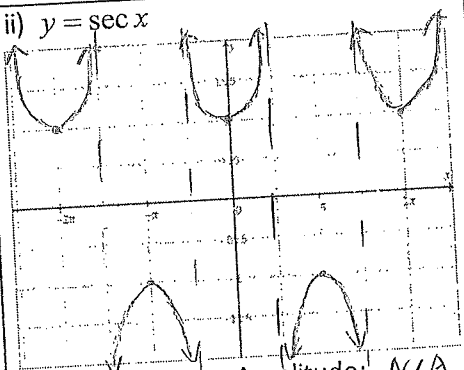
i) $y = \csc x$ $\frac{1}{\sin x}$



Period: 2π Amplitude: N/A

Domain: $x \neq (n)\pi$ Range: $-1 \geq y$ or $1 \leq y$

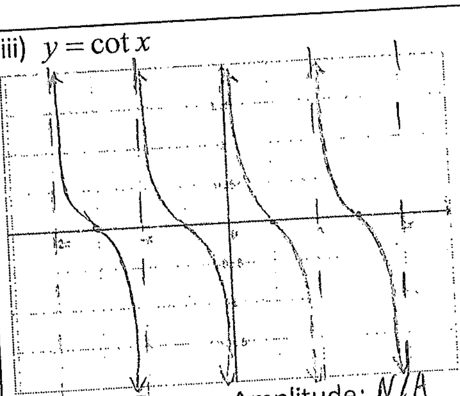
ii) $y = \sec x$



Period: 2π Amplitude: N/A

Domain: $x \neq (0.5+n)\pi$ Range: $y \geq 1$, $y \leq -1$

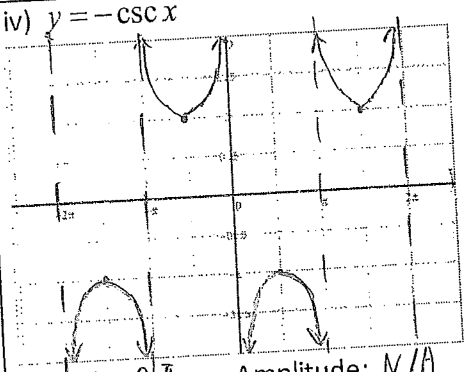
iii) $y = \cot x$



Period: π Amplitude: N/A

Domain: $x \neq (n)\pi$ Range: $y \in R$

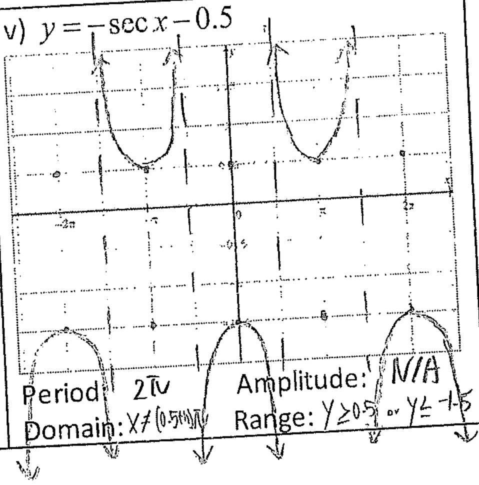
iv) $y = -\csc x$



Period: 2π Amplitude: N/A

Domain: $(n)\pi$ Range: $-1 \geq y$ or $1 \leq y$

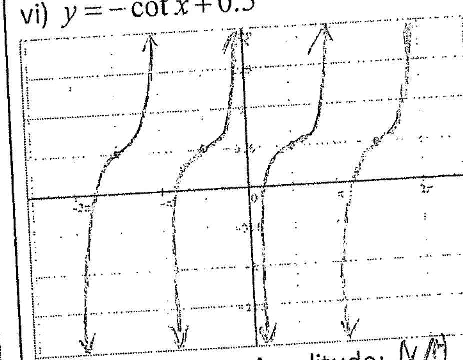
v) $y = -\sec x - 0.5$



Period: 2π Amplitude: N/A

Domain: $x \neq (0.5+n)\pi$ Range: $y \geq 0.5$ or $y \leq -1.5$

vi) $y = -\cot x + 0.5$



Period: 2π Amplitude: N/A

Domain: $x \neq n(\pi)$ Range: $y \in R$

14. Simplify the following in terms ...

a) $(\sec x\csc x-\cot x)(\sin x-\csc x)$ $\frac{\sin^2 x}{\sin x}-\frac{1}{\sin x}$ $\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)-\frac{\cos x}{\sin x}$ $\frac{1}{\cos x\sin x}-\frac{\cos^2 x(\cos x)}{\sin x(\cos x)}$ $\frac{-\cos^2}{\sin x}$ $1-\cos^2 x=\sin^2 x$ $\frac{\sin^2 x}{\cos x\sin x}=\left(\frac{\sin}{\cos}\right)\left(\frac{-\cos^2}{\sin}\right)=-\cos x$ //	b) $\dfrac{\frac{\cot x+1}{\cot x}-1}{\frac{\cot x-1}{\cot x}-1}$ -1 //
c) $\cot x+\tan x$ $\frac{\cos^2}{\cos\sin}+\frac{\sin^2}{\cos\sin}$ $=\frac{1}{\sin x\cos x}$ //	d) $\dfrac{\csc^2 x+\sec^2 x}{\csc x\sec x}$ $\dfrac{\frac{\cos^2}{\sin^2 x\cos^2 x}+\frac{\sin^2}{\cos^2 x\sin^2 x}}{\frac{1}{\sin x}\left(\frac{1}{\cos x}\right)}=\dfrac{\frac{\sin^2+\cos^2}{1}}{\frac{1}{\sin x\cos x}}$ $\frac{\sin^2 x+\cos^2 x}{}=\sin x\cos x$
e) $\sec A\sqrt{\dfrac{1-\sin^2 B\sin^2 A}{1+\cos^2 A\tan^2 B}}$ $\sin^2+\cos^2-\sin^2 B\sin^2 A$ $\sin^2+\cos^2/+\cos^2 A\times\frac{\sin B}{\cos B}$	f) $\dfrac{\sec x}{\tan x+\cot x}$ $\dfrac{\frac{1}{\cos x}}{\frac{\sin x\cos}{\cos x}+\frac{\cos x\sin}{\sin x}}$ $\frac{2\cos\sin}{\sin\cos}=\frac{1}{\cos x}\times\frac{1}{2}$ $=\frac{1}{2\cos x}$ //

4. Indicate the general formula for the vertical asymptotes of $y=\cot x$

V.A. = $n(\pi)$ //

5. Given each expression, calculate the value of θ for $0<\theta<2\pi$

a) $\csc\theta=-2$ $\frac{1}{\sin}=-2$ $\sin=-0.5$ $150°$ or $330°$ //	b) $\sec\theta=\frac{2\sqrt{3}}{3}$ $\frac{1}{\cos}=\frac{2\sqrt{3}}{3}$ $30°$ $\cos=\frac{3}{2\sqrt{3}}$ or $330°$ //	c) $\cot^2\theta=1$ $\frac{\cos^2}{\sin^2}=1$ $\cos^2=\sin^2$ $\sin=\cos$ $45°$, $225°$ //	d) $3\csc\theta=-2\sqrt{3}$ $\frac{1}{\sin}=\frac{-2\sqrt{3}}{3}$ $120°$ or $300°$ //
e) $\cot\theta=-\sqrt{3}$ $\frac{1}{\tan}=-\sqrt{3}$ $330°$, $150°$ //	f) $\sec\theta=\infty$ $\frac{1}{\cos}=\infty$ undefined or $\cos\theta=0$ $90°$, $270°$ //	g) $\cot\theta=0$ $\frac{\cos}{\sin}=0$ $\cos=0$ $90°$, $270°$ //	h) $\csc\theta=-1$ $\frac{1}{\sin}=-1$ $\sin=-1$ $270°$ //

i) $6\sec\theta=4\sqrt{3}$ $\frac{1}{\cos}=\frac{4\sqrt{3}}{6}$ 30° or 330°	j) $3\csc\theta+2\sqrt{3}=0$ $\frac{1}{\sin}=\frac{-2\sqrt{3}}{3}$ 240° or 300° //	k) $\tan\theta-\sqrt{3}=0$ $\sqrt{3}$ 60° or 240° //	L) $\sqrt{3}\sec\theta+2=0$ $\frac{1}{\cos}=\frac{-2}{\sqrt{3}}$ 150° or 210° //
m) $\cancel{2}\csc^2\theta-\cancel{1}=3$ $\frac{1}{\sin^2}=\cancel{\frac{4}{2}}\ 2$ $\sin^2=0.5$ $\pm\sqrt{0.5}$ 45° or 135° // 315° or 225° //	n) $\tan^2\theta=9$ $\tan=\pm3$ 71.57° or 251.57° // 288.43° or 108.43° //	o) $\cos^2\theta-\cos\theta-2=0$ $x^2-x-2=0$ 2 or -1 180° //	p) $\frac{-2}{\csc\theta}+\csc\theta=1$ $-2+\csc^2=\csc$ 210° or 30° 150°

6. Given that $\sin^2\theta+\cos^2\theta=1$ and $\tan^2\theta=1.25$, what is the value of $\sec^2\theta$?

$\frac{\sin^2}{\cos^2}=\frac{5}{4}$ $5x+4x=1$ $x=\frac{1}{9}$ $\frac{1}{\cos^2}=\frac{9}{4}$ //

7. If θ is an angle whose measure is not an integer multiple of $90°$, prove that $\cot\theta-\cot 2\theta=\frac{1}{\sin 2\theta}$

$\tan 90$ = undefined $\theta=2^\circ$

$\tan 180=0$

$\frac{1}{\tan\theta}-\frac{1}{\tan 2\theta}=\frac{1}{\sin 2\theta}$

$\frac{\cos\theta}{\sin\theta}-\frac{\cos 2\theta}{\sin 2\theta}=\frac{1}{\sin 2\theta}$

$\frac{1}{\csc(2\theta)}$

8. Simplify the following expression: $\frac{\sec A-\cos A}{\tan A}$

$\frac{1}{\cos}-\cos^2$ $\frac{\sin^2}{\sin}$ $\frac{\sin}{\cos}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

9. Simplify the following expression: $\frac{1+\sec A}{\tan A+\sin A}$

$\frac{1+\frac{1}{\cos}}{\frac{\sin}{\cos}+\sin}$ $\sin(\frac{1}{\cos}+1)$ $\frac{1}{\sin}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

10. Simplify the following expression: $\frac{\tan A+\cot A}{\sec A}$

$\frac{\frac{\sin^2}{\cos}+\frac{\cos^2}{\sin}}{\frac{1}{\cos}}$ $\frac{ad}{bc}$ $\frac{\sin\cos}{\cos}+\frac{\cos^2}{\sin}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

$\sin+\frac{\cos^2}{\sin}$

$\frac{1}{\sin a\cos a}$

11. Simplify the following expression: $\dfrac{1+\sin A}{1+\csc A}$

$\dfrac{1+\sin}{1+\frac{1}{\sin}} = \dfrac{1+\sin}{\left(\frac{1+\sin}{\sin}\right)}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

12. Simplify the following expression: $\dfrac{1+\sec A}{1+\cos A}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

13. Simplify the following expression: $\dfrac{1+\cot A}{1+\tan A}$

a) $\sin A$ b) $\cos A$ c) $\sec A$ d) $\csc A$ e) $\cot A$

14. Prove that both sides of the equation are equal: $\dfrac{\sin A+\cos A\cot A}{\cos A\csc A} = \sec A$

$\dfrac{\sin A + \cos A\left(\frac{\cos}{\sin}\right)}{\cos\left(\frac{1}{\sin}\right)} = \dfrac{1}{\cos}$

$\dfrac{\cancel{\frac{\cos}{\sin}}}{\sin^2 + \frac{\cos^2}{\cancel{\sin}}} = \cos$

$\dfrac{\cos}{\sin^2+\cos^2} = \dfrac{\cos}{1}$

15. Prove that both sides of the equation are equal: $\dfrac{1-\cos A}{\sin A} = \dfrac{1}{\csc A+\cot A}$

(1) $\dfrac{\sin^2+\cos^2-\cos}{\sin} = \dfrac{\sin^2+\cos^2\ (1)}{\frac{1}{\sin}+\frac{\cos}{\sin}}$ $\dfrac{1}{\frac{1+\cos}{\sin}} \rightarrow \dfrac{\sin(1+\cos)}{1+\cos}$

$\dfrac{1-\cos^2}{\sin} : \sin = \text{R.H.S}$

$\dfrac{1-\cos A}{\sin A} = \cos A$

$\dfrac{\sin^2+\cos^2-\cos}{\sin} = \cos$ $\sin + \cos^2 - \cos$

$-1 \le x \le 1$

16. Challenge: What are all the values of "x" between 0 *and* 2π that satisfy the equation?

$\left(5+2\sqrt{6}\right)^{\sin x} + \left(5-2\sqrt{6}\right)^{\sin x} = 2\sqrt{3}$

$\sqrt{25}+\sqrt{24}$ $\sqrt{25}\cdot\sqrt{24}$ $\sqrt{12}$

$\dfrac{1-\cos}{\sin} = \dfrac{1}{\csc+\cot}$

$\dfrac{\sin^2+\cos^2-\cos}{\sin} = \dfrac{1}{\frac{1}{\sin}+\frac{\cos}{\sin}}$

$= \dfrac{1}{\frac{1+\cos}{\sin}} = \dfrac{\sin}{1+\cos}$

$30°$, $150°$, $330°$, $210°$

(use of graphs / graphing calculator)

Date: 14/3/2025

Name: Cody Lee

Math 10/11 Enriched: Section 4.8 Proving and Verifying Basic Trigonometric Identities

$\sin(-x) = -\sin x$ $\qquad \tan x = \dfrac{\sin x}{\cos x}$ $\qquad \csc x = \dfrac{1}{\sin x}$ $\qquad \sec x = \dfrac{1}{\cos x}$ $\qquad \cot x = \dfrac{1}{\tan x}$

$\cos(-x) = \cos x$ $\qquad \sin^2 x + \cos^2 = 1$ $\qquad \sin 2x = 2\sin x \cos x$

1. ***Verify*** which of the following are trigonometric identities:

a) $\tan x + \cot x = \sec x \csc x$ tan 1 + cot 1 = 57.31 sec(1) × csc(1) = 57.31 trig identity //	b) $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$ trig identity // $\frac{1}{\cos(1)^2} + \frac{1}{\sin(1)^2}$ = 1.0006
c) $\sec^2 x - \csc^2 x = \dfrac{\sec^2 x}{\csc^2 x}$ not a trig identity //	d) $\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$ trig identity //
e) $\cos^2 x = \sin x(\csc x + \sin x)$ 0.97 1.03 not a trig identity //	f) $\sin^2 x = \cos x(\sec x - \cos x)$ trig identity // 0.03 0.03 $\cos\left(\frac{1}{\cos} - \cos\right)$ $\sin^2 = 1 - \cos^2$ //
g) $\sin x \tan x + \sec x = \dfrac{\sin^2 x + 1}{\cos x}$ 1.046 trig identity // 1.046 $\sin \cdot \frac{\sin}{\cos} + \frac{1}{\cos} = \frac{\sin^2 + 1}{\cos}$ R.H.S //	h) $\dfrac{\sin x + \tan x}{\cos x + 1} = \tan x$ trig identity // 0.176 0.176 $\sin + \frac{\sin}{\cos}$ $\sin\cos + \sin$

2. Simplify each of the following expressions:

a) $\sin^2 x + \cos^2 x + \cot^2 x$ $\frac{\sin^2}{\sin^2} + \frac{\cos^2}{\sin^2}$ $= \frac{1}{\sin^2} = \csc^2 x$ //	b) $\dfrac{\sin 2x}{1 + \cos 2x}$ $\frac{2\sin x \cos x}{\cos^2 x + \cos^2 x} = \frac{\sin \cos}{\cos^2}$ $= \tan x$ //

$\sin(2x+x) = \sin 2x \cos x + \sin x \cos 2x$ $= 2\sin x \cos x \cos x + \sin x (\cos^2 x - \sin^2 x)$ $= 2\sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x$ $= \sin x (2\cos^2 x + \cos^2 x - \sin^2 x)$ $= \sin x (3\cos^2 x - \sin^2 x)$	
c) $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x}$ $\dfrac{\sin x(3\cos^2 x - \sin^2 x)}{\sin x}$ $(3\cos^2 x - \sin^2 x) - (3\sin^2 x - \cos^2 x)$ $= (\cos^2 x - 3\sin^2 x)$ $2\cos^2 x + 2\sin^2 x$ $= 2$ //	d) $\cos(a+b)\cos b + \sin(a+b)\sin b$ $(\cos a \cos b - \sin a \sin b)\cos b$ $+$ $(\sin a \cos b + \sin b \cos a)\sin b$ $\cos(a)\cos^2(b) - \cancel{\sin(a)\sin(b)\cos(b)} + \cancel{\sin(a)\sin(b)\cos(b)} +$ $\cos(a)\cos^2(b) + \sin^2(b)\cos(a)$ $= \cos(a)(\sin^2 b + \cos^2 b) \leftarrow 1 = \cos(a)$ //
e) $\sin\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)\sin\left(\frac{\pi}{3}+x\right)$ $\left(\sin\left(\frac{\pi}{3}\right)\cos(x) - \sin(x)\cos\left(\frac{\pi}{3}\right)\right)$ $\cos a \cos b + \sin a \sin b +$ $\sin\left(\frac{\pi}{3}+x+\frac{\pi}{3}-x\right)$ $\cos a \cos b - \sin a \sin b$ $2 \cos a \cos b$ $\sin\left(\frac{2\pi}{3}\right)$ $= \frac{\sqrt{3}}{2}$ //	f) $\dfrac{\cos^3 x - \cos x}{\sin^3 x}$ $\quad \sin^2 + \cos^2 = 1$ $\cos(\cos^2 - 1) = \dfrac{-\sin^2}{\sin^3}$ $\dfrac{\cos}{\sin x}$ $= -\cot x$ //

3. Suppose $0 < x < 90°$ and $2\sin^2 x + \cos^2 x = \frac{25}{16}$. What is the value of $\sin x$?

$\sin^2 x = \frac{9}{16} = \pm\frac{3}{4}$ $\quad 48.59°$ //

4. Evaluate the following: $\sin\left(\dfrac{\pi}{6}\right) + \underbrace{\sin^2\left(\dfrac{\pi}{6}\right) + \cos^2\left(\dfrac{\pi}{6}\right)}_{1}$

$\sin 30$
$= 0.5$ $\quad 1.5$ //

5. Determine the value of $\sin^2\left(\dfrac{\pi}{8}\right) + \cos^2\left(\dfrac{3\pi}{8}\right) + \sin^2\left(\dfrac{5\pi}{8}\right) + \cos^2\left(\dfrac{7\pi}{8}\right)$

$\sin(\pi - \theta) = \sin\theta$

$\sin^2\left(\frac{7\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \sin^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{8}\right)$

$= 2$ //

6. Suppose that, for some angles "x" and "y" $\sin^2 x + \cos^2 y = \dfrac{3a}{2}$ and $\cos^2 x + \sin^2 y = \dfrac{1}{2}a^2$, determine the possible value(s) of "a".

$\sin^2 x + \sin^2 y + \cos^2 x + \cos^2 y = 2$

$\frac{3a}{2} + \frac{a^2}{2} = 2$

$3a + a^2 = 4$

$a^2 + 3a - 4 = 0$

$a = 1$ or -4 // //

7. What is the sum of all values of all values of "x" between 0 and 2π inclusive that satisfy the equation:

$\tan x+1=\sec^2 x$?

$= \frac{1}{\cos^2}$

$\frac{\sin}{\cos} + \frac{\cos}{\cos} = \frac{\sin(x)+\cos(x)}{\cos(x)} = \frac{1}{\cos^2(x)}$

8. If $\cos(x)=\frac{3}{4}\frac{A}{H}$ and "x" is in the first quadrant, what is the value of $\sin(2x)$?

$16-9=7$

$\frac{\sqrt{7}}{4}=\sin x$

$\sin 2x = 2\sin x \cos x$

$\frac{\sqrt{7}}{4} \times \frac{3}{4} = \frac{3\sqrt{7}}{16} \times 2$

$= \frac{3\sqrt{7}}{8}$ //

9. Suppose that $\sin a+\sin b=\sqrt{\frac{5}{3}}$ and $\cos a+\cos b=1$. What is the value of $\cos(a-b)$?

$\sin a = \sqrt{\frac{5}{3}} - \sin b$

$\left(\frac{\sqrt{5}}{3} - \sin b\right)(\sin b)$

$= \frac{\sqrt{5}}{3}\sin b - \sin^2 b = \frac{\sqrt{5}}{3}\sin b + \cos^2 b - 1$

$\cos a = 1-\cos b$

$\cos a\cos b + \sin a \sin b$

$(1-\cos b)(\cos b)$

$= \cos b - \cos^2 b$

$\frac{1}{3}$ //

$\cos^2 a + \cos^2 b + 2\cos a \cos b = 1$

$\sin^2 a + \sin^2 b + 2\sin a \sin b = \frac{5}{3}$

10. In degrees, what are all ordered pairs of angles (x,y) for which both angles are between 0 and $90°$ and satisfy the equation $\sin^2 x+\sin^2 y=\sin x+\sin y$?

$(90, 90)$ $(0, 0)$

$\sin^2 x + \sin^2 y + 2\sin x \sin y = (\sin x + \sin y)^2$

$\sin^4 x + \sin^4 y + 2\sin^2 x \sin^2 y$

$= \sin^2 x + \sin^2 y + 2\sin x \sin y$

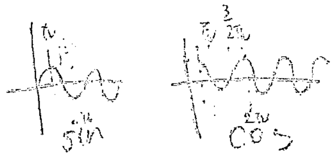
$(\sin x + \sin y)^2 - 2\sin x \sin y$

$= \sin^2 x + \sin^2 y$

$a^2+b^2 = a+b$

11. In degrees, what are all values of 'x" between 0 *and* $360°$ for which $\sin x>\sqrt{1-\sin^2 x}$?

$\sin x > \cos x$



$\frac{\pi}{2}$

$45°$ to $225°$ //

12. What are the degree measures of all positive angles between 0 and $90°$ which satisfy the equation:

$\sin^2 x+\cos^2 x+\tan^2 x+\cot^2 x+\sec^2 x+\csc^2 x=31$

$1 + \tan^2 x + \cot^2 x + 1 - 1$

$2\cdot(\sec^2 x + \csc^2 x) = \cancel{32}\ 16$

$\frac{\sin^2}{\sin^2\cos^2} + \frac{\cos^2}{\sin^2\cos^2} = 16$

$\frac{\sin^2+\cos^2}{\sin^2\cos^2} = 16$

$\frac{1}{16} = \sin^2\cos^2$

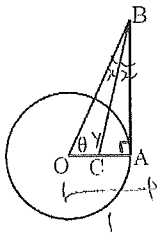
$\sin x \cos x = \frac{1}{4}$

15 / or 75 //

13. A circle centered at "O" has radius 1 and contains the point A. Segment AB is tangent to the circle at "A" and $\angle AOB = \theta$. If point "C" lies on $\overline{OA}$ and $\overline{BC}$ bisects $\angle ABO$, then what is the length of OC?

$\frac{1}{\cos^2}$

a) $\sec^2\theta - \tan\theta$ b) $\frac{1}{2}$ c) $\frac{\cos^2\theta}{1+\sin\theta}$ d) $\frac{1}{1+\sin\theta}$ e) $\frac{\sin\theta}{\cos^2\theta}$



$\frac{BA}{BO} = \sin\theta$ $\frac{BA}{AO} = \tan\theta$

$\frac{\sin\theta}{BC} = \frac{\sin y}{BO} = \frac{\sin x}{OC}$

14. If $\cos\theta = 2\tan\theta$, solve for the numerical value of $\cos^2\theta$.

$1 - \sin^2 = ?$

$\frac{2\sin}{\cos} = \cos$ $\cos^2 = 2\sin x$

$1 - \sin^2 = 2\sin x$

15. Simplify: $\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) - \sin\left(\frac{\pi}{6}+x\right)\sin\left(\frac{\pi}{6}-x\right)$

$\cos(a+b)$

$\cos\left(\frac{2\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right)$ //

16. If $\sin x = \frac{-1}{3}$ and "x" is in quadrant 3, then what is the value of $\sin 2x$?

$2\sin x \cos x$

$-\frac{\sqrt{8}}{3} \times \frac{-1}{3} = \frac{2\sqrt{8}}{3}$ //

17. What is the value of $\sin(a+b)$ if $\sin a = \frac{-3}{5}$ and $\cos b = \frac{3}{5}$, with both "a" and "b" in the fourth quadrant.

$\sin a \cos b + \sin b \cos a$

$-\frac{3}{5} \times \frac{3}{5} + 2\ -\frac{4}{5} \times \frac{4}{5} = \frac{-9}{25} + \frac{-16}{25}$ S A T C

$= -1$ //

18. Simplify the expression: $(\sin x - \cos x)^2 - (\sin x + \cos x)^2$

$a^2 - b^2$

a) 0 b) $-\sin 2x$ c) $\sin 2x$ d) $-2\sin 2x$

$(a+b)(a-b)$

$\sin - \cos - \sin - \cos$

$= -2\cos$

$(\sin - \cos + \sin + \cos)$

$= 2\sin$

$= -4\cos\sin$

19. Challenge: Evaluate and simplify the following without a calculator: $(\cos 36°)(\cos 108°)$

$\cos^4(36) - 3\sin^2(36)\cos^2(36)$

$(\cos x)(\cos 3x)$

$(\cos 2x + x)$

$\cos x(\cos 2x \cos x - \sin 2x \sin$

$\cos x\left[(\cos^2 x - \sin^2 x)\cos x - (2\sin x\cos\right.$

$\sin$

$\cos^3 x - \cos x\sin^2 x - 2\sin^2 x\cos x$

(a) Suppose that, for some angles x and y,

$$\begin{aligned}\sin^2 x + \cos^2 y &= \tfrac{3}{2}a \\ \cos^2 x + \sin^2 y &= \tfrac{1}{2}a^2\end{aligned}$$

Determine the possible value(s) of a.

1 , -4

c) $\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}$	d) $\cos(a+b)\cos b+\sin(a+b)\sin b$
e) $\sin\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)\sin\left(\frac{\pi}{3}+x\right)$	f) $\frac{\cos^3 x-\cos x}{\sin^3 x}$

3. Suppose $0<x<90^\circ$ and $2\sin^2 x+\cos^2 x=\frac{25}{16}$. What is the value of $\sin x$?

4. Evaluate the following: $\sin\left(\frac{\pi}{6}\right)+\sin^2\left(\frac{\pi}{6}\right)+\cos^2\left(\frac{\pi}{6}\right)$

5. Determine the value of $\sin^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\sin^2\left(\frac{5\pi}{8}\right)+\cos^2\left(\frac{7\pi}{8}\right)$

6. Suppose that, for some angles "x" and "y" $\sin^2 x+\cos^2 y=\frac{3a}{2}$ and $\cos^2 x+\sin^2 y=\frac{1}{2}a^2$, determine the possible value(s) of "a".

Name: Cody Lee Date: 3/14/2025

M10/11H HW Section 4.9 P1 Sum and Difference Identities

$\sin(a-b)=\sin a\cos b-\sin b\cos a$ $\cos(a+b)=\cos a\cos b-\sin b\sin a$ $\sin(a+b)=\sin a\cos b+\sin b\cos a$

$\cos(a-b)=\cos a\cos b+\sin b\sin a$ $\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a\tan b}$ $\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$

1. Using two of any of these angles: $30°,\ 45°,\ 60°,\ 90°,\ 180°,\ 270°,$ and $360°$, add or subtract them to see how many angles that you can create: ie: $75°=30°+45°$

15, 30, 60, 120, 240, 330, 90, 180, 270, 75, 90, 120, 210, 300, 390, 105, 135, 225, 315, 405, 150, 240, 330, 420, 210, 360, 450, 540

20 f

27

2. Find the exact ratio of sine and cosine of the following angles:
$30°,\ 45°,\ 60°,\ 90°,\ 135°,\ 150°,\ 210°,\ 225°,\ 240°,\ 300°, 315°,$ and $330°$

sin	$\frac{1}{2}$	$\frac{1}{\sqrt2}$	$\frac{\sqrt3}{2}$	1	$\frac{1}{\sqrt2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt2}$	$-\frac{\sqrt3}{2}$	$-\frac{\sqrt3}{2}$	$\frac{1}{\sqrt2}$	$-\frac{1}{2}$
cos	$\frac{\sqrt3}{2}$	$\frac{1}{\sqrt2}$	$\frac{1}{2}$	0	$-\frac{1}{\sqrt2}$	$-\frac{\sqrt3}{2}$	$-\frac{\sqrt3}{2}$	$-\frac{1}{\sqrt2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{\sqrt2}$	$\frac{\sqrt3}{2}$
tan	$\frac{1}{\sqrt3}$	1	$\sqrt3$	undefined	-1	$-\frac{1}{\sqrt3}$	$\frac{1}{\sqrt3}$	1	$\sqrt3$	$-\sqrt3$	-1	$-\frac{1}{\sqrt3}$

3. If we are given the ratio $\sin a=\frac{3}{7}$, what is the value of $\cos a=$??

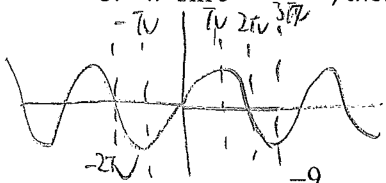
$\pm\frac{\sqrt{40}}{7}$

S A
T C

4. If $\cos Q=\frac{-3}{11}$ and angle Q is in quadrant 3, then what is the value of $\sin Q$??

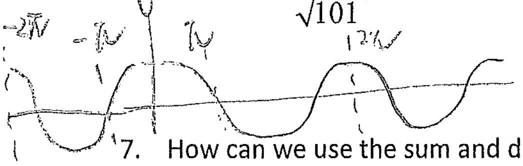
$-\frac{\sqrt{112}}{11}$

5. IF $\sin\theta=\frac{-\sqrt5}{7}$, then what is the value of $\sin(-\theta)$ equal to? Explain



$\frac{\sqrt5}{7}$, sin graph is an odd function

6. IF $\cos\theta=\frac{-9}{\sqrt{101}}$, then what is the value of $\cos(-\theta)$ equal to? Explain



$\frac{-9}{\sqrt{101}}$, cos function is an even function

7. How can we use the sum and difference identities to find the exact value of $\sin 75°$? Explain:

$\frac{\sqrt2+\sqrt6}{4}$

$\sin(30+45)$
$=\sin a\cos b+\sin b\cos a$
$\sin 30\times\cos 45+\sin 45\times\cos 30$
$\frac{1}{2}\times\frac{\sqrt2}{2}$, $\frac{\sqrt2}{2}\times\frac{\sqrt3}{2}$

8. Is the following an identity? $\cos(a-b)=\cos a-\cos b$? Explain:

9. Given that $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, what is $\tan(a+b)=??$ What is $\tan(a-b)=??$

10. Find the exact value of each trigonometric expression:

a) $\sin(45°+30°)=$	b) $\cos\left(\dfrac{3\pi}{4}+\dfrac{2\pi}{3}\right)$
c) $\cos(120°+45°)$	d) $\sin(-15°)$
e) $\cos\left(\dfrac{5\pi}{12}\right)$	f) $\cos(255°)$
g) $\tan(300°-45°)$	h) $\tan(300°+45°)$

8. Is the following an identity? $\cos(a-b) = \cos a - \cos b$? Explain:

no verify: $\cos(90-45) = \cos 90 - \cos 45$

$0 - \frac{1}{\sqrt{2}} \neq \frac{1}{\sqrt{2}}$ //

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

9. Given that $\tan\theta = \frac{\sin\theta}{\cos\theta}$, what is $\tan(a+b) = ??$ What is $\tan(a-b) = ??$

$\frac{\sin(a+b)}{\cos(a+b)} = \frac{\sin a \cos b + \sin b \cos a}{\cos a \cos b - \sin a \sin b}$ //

$\frac{\sin(a-b)}{\cos(a-b)} = \frac{\sin a \cos b - \sin b \cos a}{\cos a \cos b + \sin a \sin b}$ //

10. Find the exact value of each trigonometric expression:

a) $\sin(45°+30°) =$ $\frac{\sqrt{2}+\sqrt{6}}{4}$ (same as Q.7)	b) $\cos\left(\frac{3\pi}{4}+\frac{2\pi}{3}\right)$ 135° 120 225 +30 $\cos 225 \times \cos 30 - \sin 225 \sin 30$ $-\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - -\frac{1}{\sqrt{2}} \times \frac{1}{2}$ $\frac{-\sqrt{3}}{2\sqrt{2}} - \frac{-1}{2\sqrt{2}}$ $= \frac{1-\sqrt{3}}{2\sqrt{2}}$ //
c) $\cos(120°+45°)$ 210 - 45 $\cos 210 \cos 45 - \sin 210 \sin 45$ $\frac{-\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \left(-\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$ $\frac{-\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$ $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ //	d) $\sin(-15°)$ $= \sin(45-30) \times -1$ $-1\left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}}$ //
e) $\cos\left(\frac{5\pi}{12}\right)$ (45+30) $\cos 45 \cos 30 - \sin 45 \sin 30$ $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $\frac{\sqrt{3}-1}{2\sqrt{2}}$ //	f) $\cos(255°)$ $\frac{1-\sqrt{3}}{2\sqrt{2}}$ (same as 10 b) //
g) $\tan(300°-45°)$ tan 255 $\sin 300 \cos 45 - \sin 45 \cos 300$ $-\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{-\sqrt{3}-1}{2\sqrt{2}}$ $\frac{-\sqrt{3}-1}{1-\sqrt{3}}$ //	h) $\tan(300°+45°)$ $\frac{\sin 300 \cos 45 + \cos 300 \sin 45}{\cos 300 \cos 45 - \sin 300 \sin 45}$ $\frac{-\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$ $\frac{1-\sqrt{3}}{2\sqrt{2}}$ $\frac{-\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$ $1+\sqrt{3}$ $\frac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \frac{(1-\sqrt{3})}{(1-\sqrt{3})}$ $1-2\sqrt{3}+3$ $= \frac{4-2\sqrt{3}}{-2} = \boxed{\sqrt{3}-2}$ //
i) $\cos 195°$ $\cos(225-30)$ $-\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + -\frac{1}{\sqrt{2}} \times \frac{1}{2}$ $-\sqrt{3}-1$ $\frac{-\sqrt{3}-1}{2\sqrt{2}}$ //	j) $\tan\left(\frac{\pi}{12}\right)$ tan(45-30) $\frac{\sqrt{3}-1}{\sqrt{3}+1} \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$ $\frac{4-2\sqrt{3}}{2}$ $\boxed{2-\sqrt{3}}$ // $\cos 45 \cos 30 + \sin 45 \sin 30$ $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$
k) $\sin\left(\frac{5\pi}{12}\right)$ $\sin(75) = \frac{\sqrt{2}+\sqrt{6}}{4}$ (same as Q.7) //	L) $\sin 375°$ $\sin(315+60)$ $\frac{1}{\sqrt{2}} \times \frac{1}{2} + -\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{-2\sqrt{2}}$ $\boxed{\frac{\sqrt{3}-1}{-2\sqrt{2}}}$ //

Name:_______________ Date:_______________

M10/11H HW Section 4.9 P1 Sum and Difference Identities

$\sin(a-b) = \sin a\cos b - \sin b\cos a$ $\quad\cos(a+b) = \cos a\cos b - \sin b\sin a$ $\quad\sin(a+b) = \sin a\cos b + \sin b\cos a$

$\cos(a-b) = \cos a\cos b + \sin b\sin a$ $\quad\tan(a+b) = \dfrac{\tan a + \tan b}{1-\tan a\tan b}$ $\quad\tan(a-b) = \dfrac{\tan a - \tan b}{1+\tan a\tan b}$

1. Using two of any of these angles: 30°, 45°, 60°, 90°, 180°, 270°, and 360°, add or subtract them to see how many angles that you can create? ie. 75° = 30° + 45°

2. Find the exact ratio of sine and cosine of the following angles:
   30°, 45°, 60°, 90°, 135°, 150°, 210°, 225°, 240°, 300°, 315°, and 330°

3. If we are given the ratio $\sin a = \dfrac{3}{7}$, what is the value of $\cos a = ??$

4. If $\cos Q = \dfrac{-3}{11}$ and angle Q is in quadrant 3, then what is the value of $\sin Q$??

5. IF $\sin\theta = \dfrac{-\sqrt{5}}{7}$, then what is the value of $\sin(-\theta)$ equal to? Explain

6. IF $\cos\theta = \dfrac{-9}{\sqrt{101}}$, then what is the value of $\cos(-\theta)$ equal to? Explain

7. How can we use the sum and difference identities to find the exact value of $\sin 75°$? Explain:

$\sin 270 = -1$

$\sin 0 = 0$

11. Simplify each expression to an exact form:

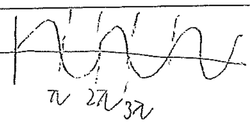
a) $\cos(\theta + 2\pi)$

$+360°$

$\cos(\theta)$ //

b) $\sin\left(\theta - \frac{3\pi}{2}\right)$

$-270°$



$\cos\theta$ //

c) $\tan\left(\theta + \frac{\pi}{2}\right)$

$90°$

$\frac{\sin\theta\cos 90 + \sin 90\cos\theta}{\cancel{\cos\theta\cos 90} - \sin\theta\sin 90}$

$= -\frac{\cos\theta}{\sin\theta} = -\cot\theta$ //

d) $\cos(\theta - 2\pi)$

$-360°$

$\cos\theta$ //

a) $\cos 70°\cos 20° + \sin 70°\sin 20°$

$\cos(70 - 20) = \cos(50)$ //

b) $\sin 20°\cos 10° + \cos 20°\sin 10°$

$\sin 30$ //

or $\frac{1}{2}$ //

c) $\cos 110°\cos 10° - \sin 110°\sin 10°$

$\cos(120)$ //

or

$-\frac{1}{2}$ //

d) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \cos\frac{\pi}{12} = \cos 15$ //

e) $\sin 42°\cos 17° - \cos 42°\sin 17°$

$\sin 25$ //

f) $\dfrac{\tan\left(\frac{\pi}{9}\right) + \tan\left(\frac{5\pi}{36}\right)}{1 - \tan\left(\frac{\pi}{9}\right)\tan\left(\frac{5\pi}{36}\right)}$

$\frac{\tan 20 + \tan 25}{1 - \tan 20 \tan 25}$

$\tan a+b$

$\frac{4\pi}{36} + \frac{5\pi}{36} = \frac{9\pi}{36}$

$= \tan\frac{\pi}{4}$ or $\tan 45$ (1) //

g) $\sin\left(\frac{\pi}{3} - x\right)\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right)\sin\left(\frac{\pi}{3} + x\right)$

$\sin\left(\frac{\pi}{3} - \cancel{x} + \frac{\pi}{3} + \cancel{x}\right)$

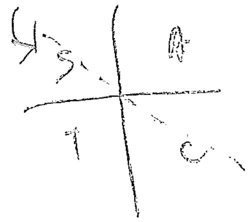
$= \sin\frac{2\pi}{3} = \sin 120$ //

H) $\sin a\cos 2a + \cos a\sin 2a$

$= \sin(3a)$ //

12. If $\sin a = \frac{4}{9}$, $90° < a \leq 180°$, then what is the exact value of $\sin(a+180°)$ equal to?

$-\frac{4}{9}$ //



13. Given that $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, and $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$ find the exact value of $\cos 75°$, $\sin 75°$ $\sin 15°$, and $\cos 15°$.

$\cos 75$

$\cos(30+45)$

$\cos 30 \cos 45 - \sin 30 \sin 45$

$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$\cos 75 = \frac{\sqrt{3}-1}{2\sqrt{2}} = \sin 15$ //

$\sin 75 = \cos 15$

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$ //

14. Given that $\sin a = \frac{3}{5}$ and $\cos b = \frac{4}{7}$, where both angles "a" and "b" are in quadrant 1, then find the following:

$\cos a = \frac{4}{5}$ $\quad \sin b = \frac{\sqrt{33}}{7}$ $\quad 49 - 16 = 33$

$\sin(a+b)$, $\sin(a-b)$, $\cos(a+b)$, and $\cos(a-b)$

$= \sin a \cos b + \sin b \cos a$

$= \frac{3}{5} \times \frac{4}{7} + \frac{\sqrt{33}}{7} \times \frac{4}{5}$

$\frac{12}{35} + \frac{4\sqrt{33}}{35} = \frac{12+4\sqrt{33}}{35}$ //

$= \sin a \cos b - \sin b \cos a$

$= \frac{12 - 4\sqrt{33}}{35}$ //

$= \cos a \cos b - \cos a \sin b$

$= \frac{4}{7} \times \frac{4}{5} - \frac{3}{5} \times \frac{\sqrt{33}}{7}$

$\frac{16 - 3\sqrt{33}}{35}$ //

$= \cos a \cos b + \sin a \sin b$

$= \frac{16 + 3\sqrt{33}}{35}$ //

15. Simplify the following: $\cos(x+\frac{\pi}{2}) - \cos(x-\frac{\pi}{2})$

a) 0 b) $-2\sin x$ c) $2\sin x$ d) $2\cos x$ e) $-2\cos x$

$(\cancel{\cos x \cos \frac{\pi}{2}} - \sin x \sin \frac{\pi}{2}) - (\cancel{\cos x \cos \frac{\pi}{2}} + \sin x \sin \frac{\pi}{2})$

$-2 \sin x \sin 90$

1

16. Suppose $\cos x = 0$ and $\cos(x+z) = 0.5$. What is the smallest possible positive value of "z"?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{5\pi}{6}$ (E) $\frac{7\pi}{6}$

$z = 30$

$\cos 270 = 0$

$\cos 300 = 0.5$

$\cos 60 = 0.5$

17. Suppose that $\sin a + \sin b = \sqrt{5/3}$ and $\cos a + \cos b = 1$. What is $\cos(a-b)$

$\cos a \cos b + \sin a \sin b$

$\frac{5}{3} = \sin^2 a + 2\sin a \sin b + \sin^2 b$

$1 = \cos^2 a + 2\cos a \cos b + \cos^2 b$

$\frac{5}{3} + \frac{3}{3} - 2 = 2 \sin a \sin b + \cos a \cos b$